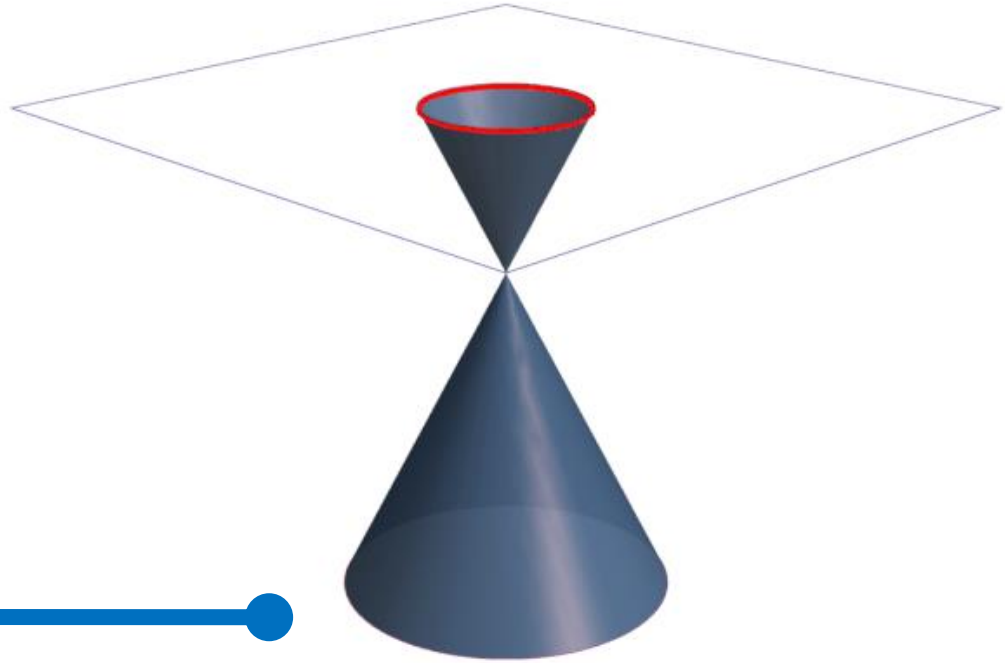
Profesor:
Jonathan Cumpa Velásquez

PITAGORAS
ACADEMIA

TRIGONOMETRÍA

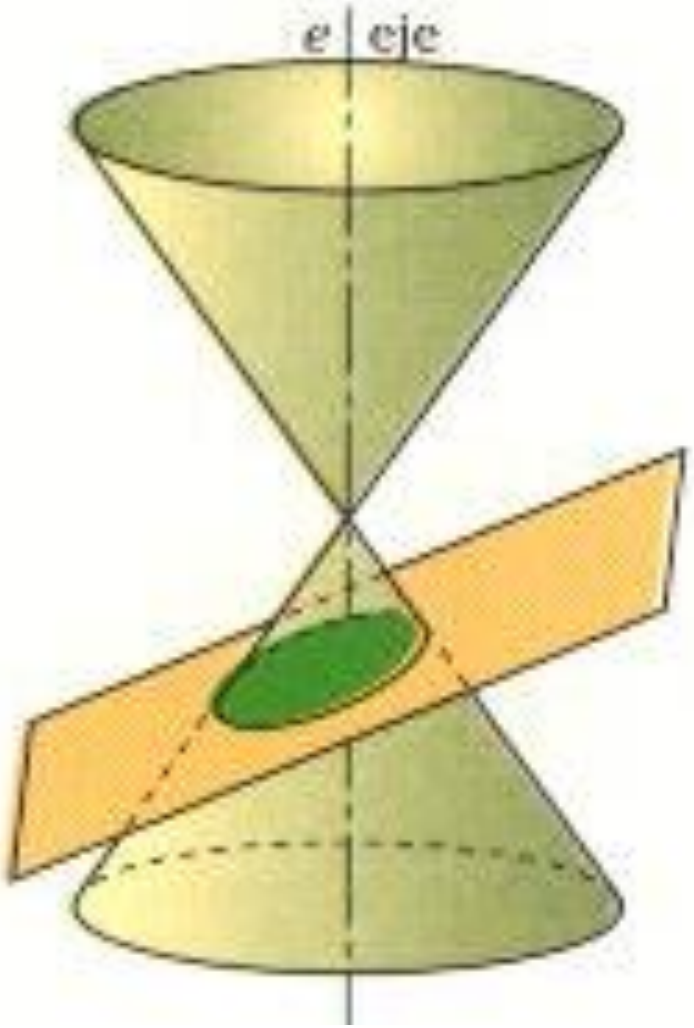
GRUPO PITÁGORAS

CÓNICAS



ELIPSE

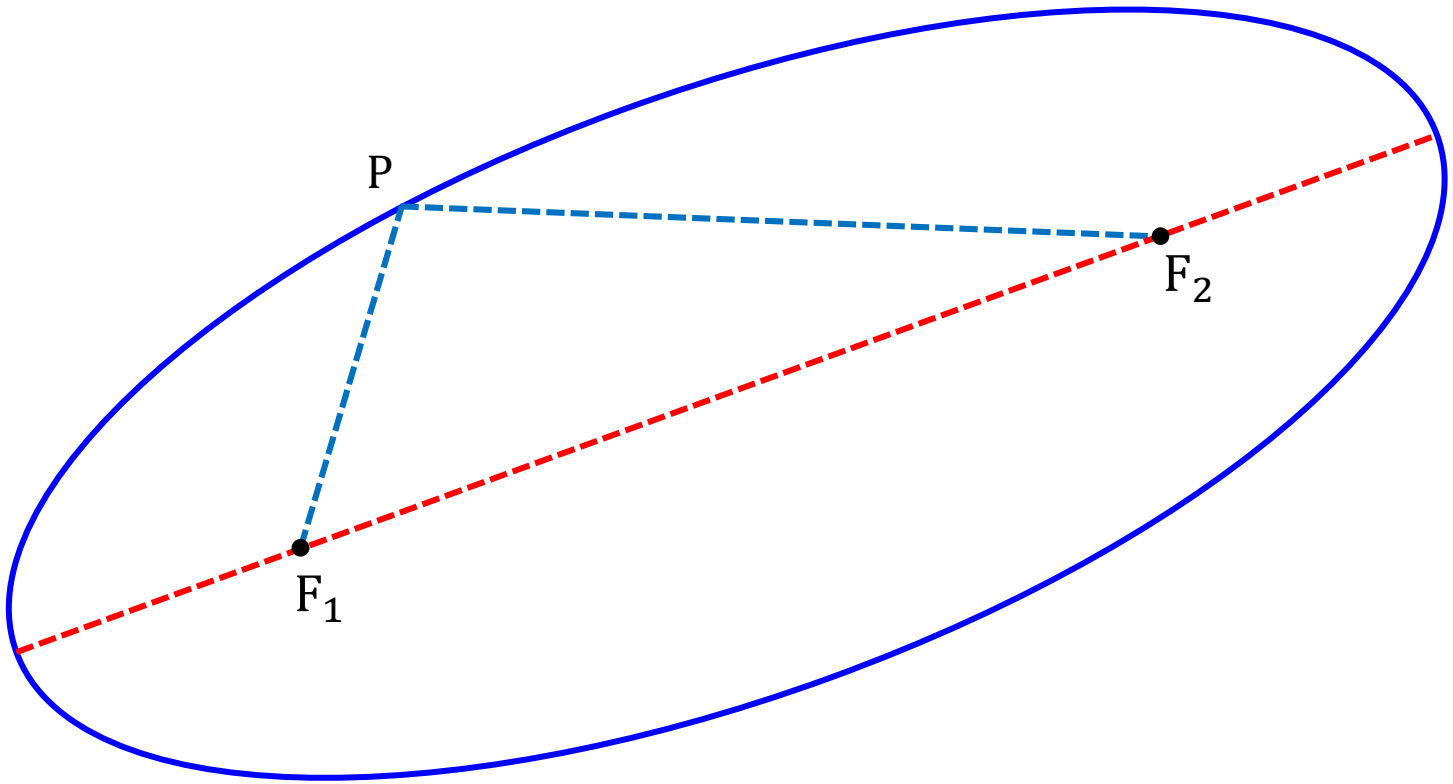
ELIPSE



ELIPSE

1. DEFINICIÓN:

Es el lugar geométrico de todos los puntos que pertenecen a un mismo plano, tales que la suma de sus distancias hacia dos puntos fijos del plano, llamados focos, es constante.

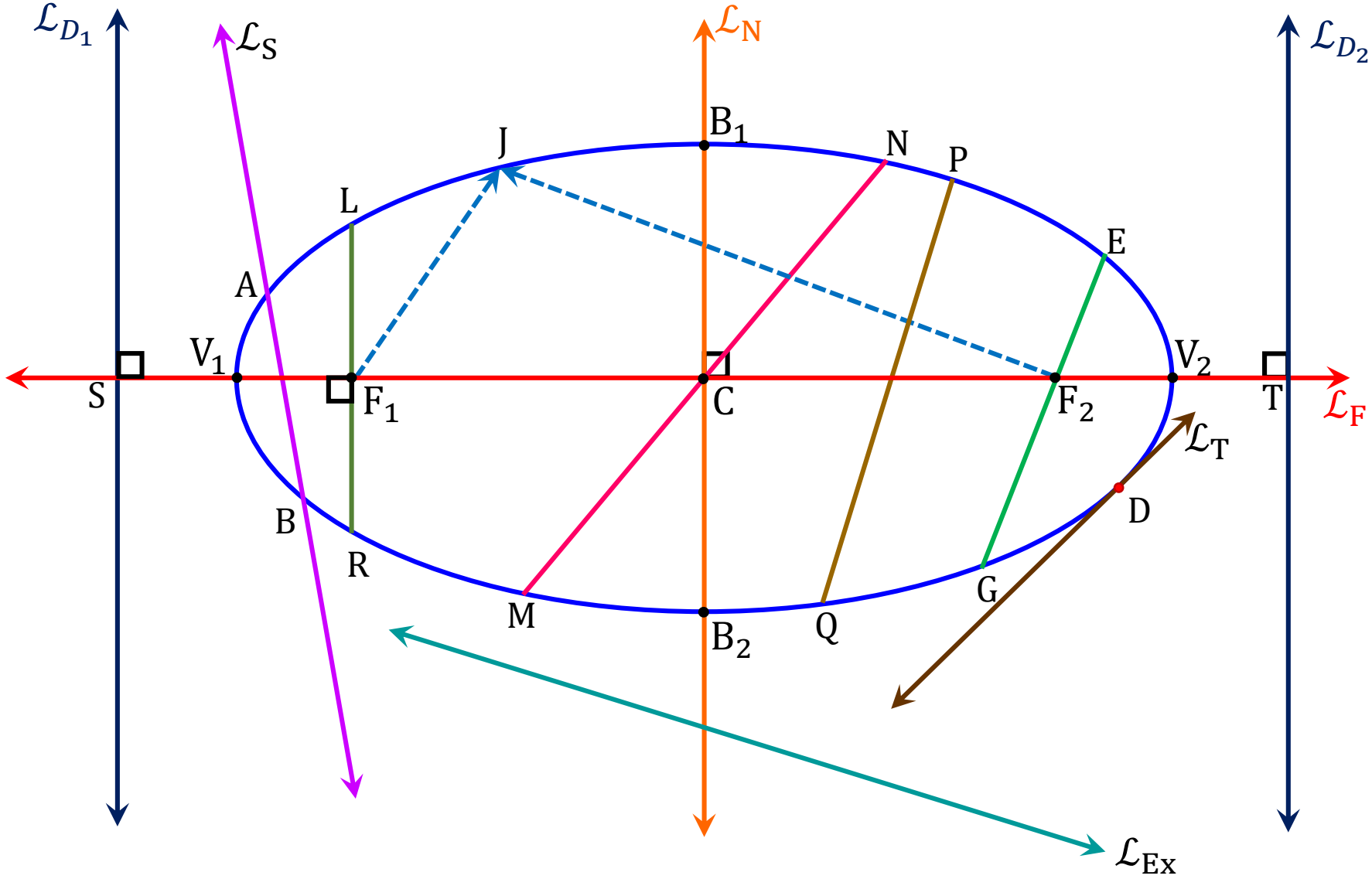


$$d\overline{PF_1} + d\overline{PF_2} = cte.$$

$$\overline{PF_1} + \overline{PF_2} = 2a$$

ELIPSE

2. ELEMENTOS:



C: Centro

$\mathcal{L}_F$: Eje Focal

$\mathcal{L}_N$: Eje Normal

$\mathcal{L}_{D_1}$ y $\mathcal{L}_{D_2}$: Directrices

V_1 y V_2: Vértices

F_1 y F_2: Focos

$\overline{PQ}$: Cuerda

$\overline{EG}$: Cuerda Focal

$\overline{MN}$: Diámetro

$\overline{LR}$: Lado Recto

F_1J y F_2J: Radios Focales

$\overline{ST}$: Distancia entre Directrices

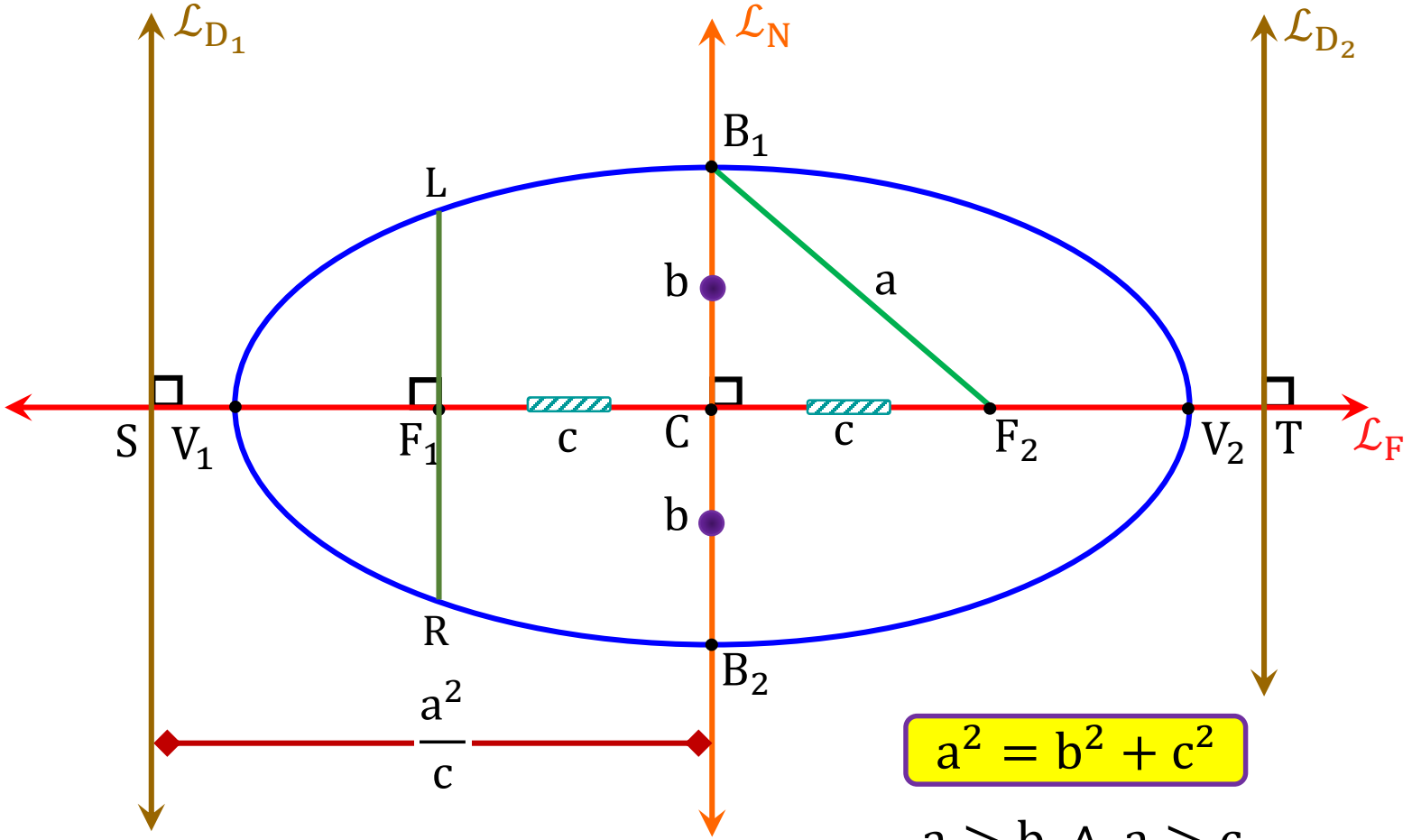
$\mathcal{L}_T$: Recta Tangente

$\mathcal{L}_S$: Recta Secante

$\mathcal{L}_{Ex}$: Recta Exterior

ELIPSE

3. RELACIONES FUNDAMENTALES:

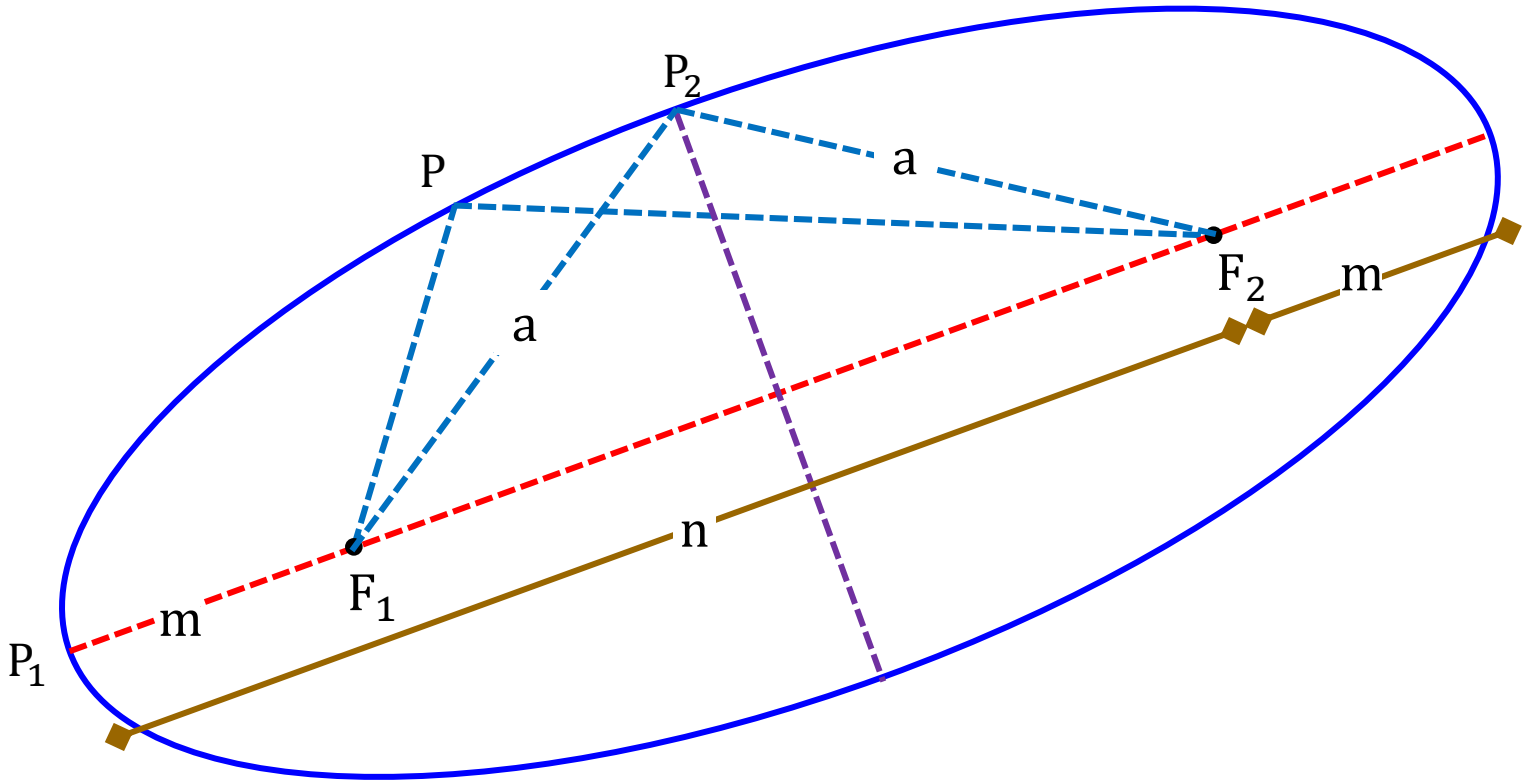


$$a^2 = b^2 + c^2$$

$a > b \wedge a > c$

- $\overline{V_1 V_2}$: Eje mayor $= 2a$
- $\overline{B_1 B_2}$: Eje menor $= 2b$
- $\overline{F_1 F_2}$: Segmento focal $= 2c$
- $\overline{LR} = \frac{2b^2}{a}$
- $e = \frac{c}{a}$, $e < 1$
- $\overline{ST} = \frac{2a}{e} = \frac{2a^2}{c}$

ELIPSE



$$d\overline{PF_1} + d\overline{PF_2} = cte.$$

$$\overline{PF_1} + \overline{PF_2} = 2a$$

$$m + n = 2a$$

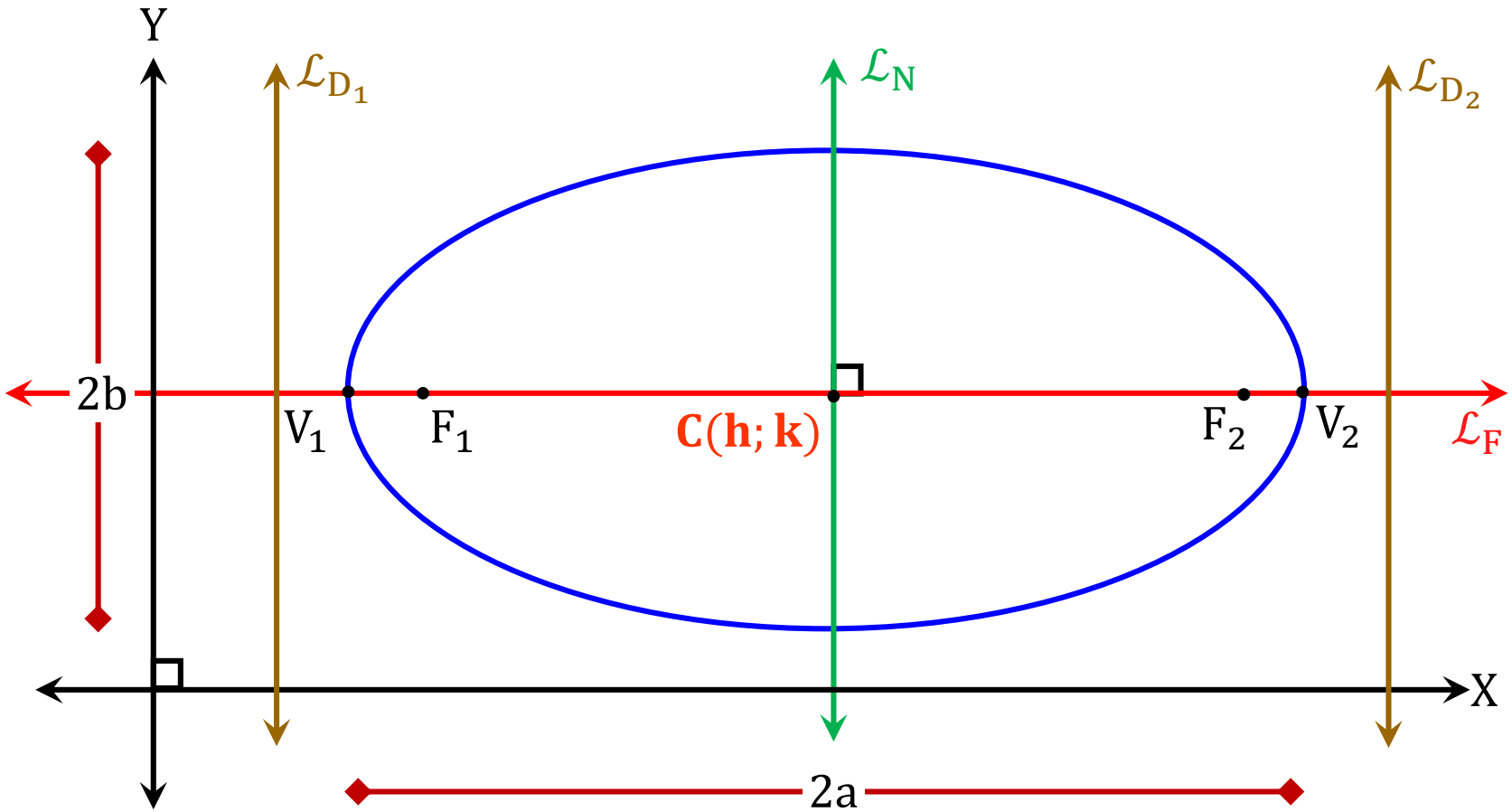
ELIPSE

4. ECUACIÓN DE LA ELIPSE:

4.1. Eje Focal paralelo al eje X:

a) Ecuación Ordinaria:

Centro en C(h; k)

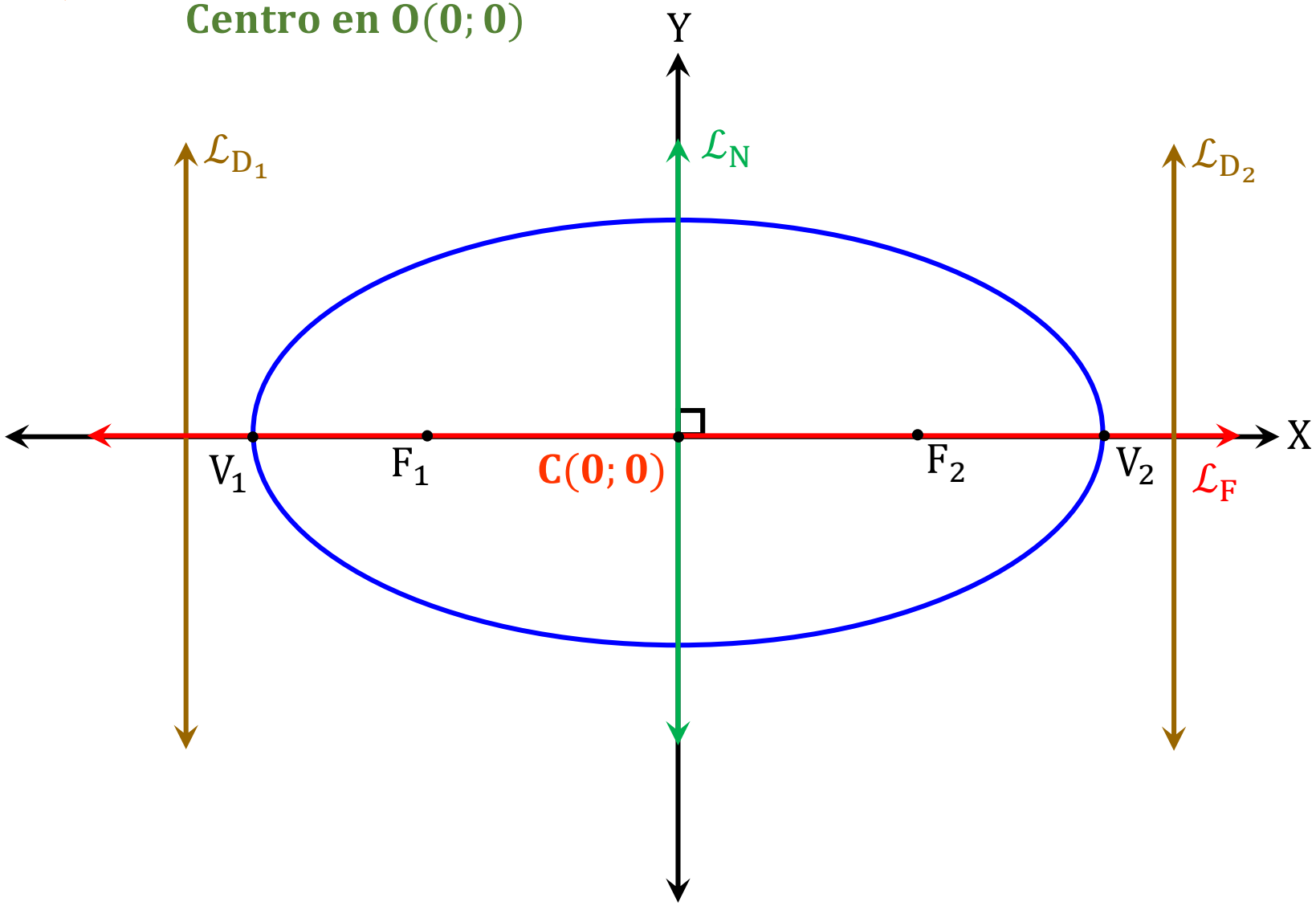


$$\mathcal{E}: \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

ELIPSE

b) Ecuación Canónica:

Centro en O(0; 0)



$$\mathcal{E}: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ELIPSE

c) Ecuación General:

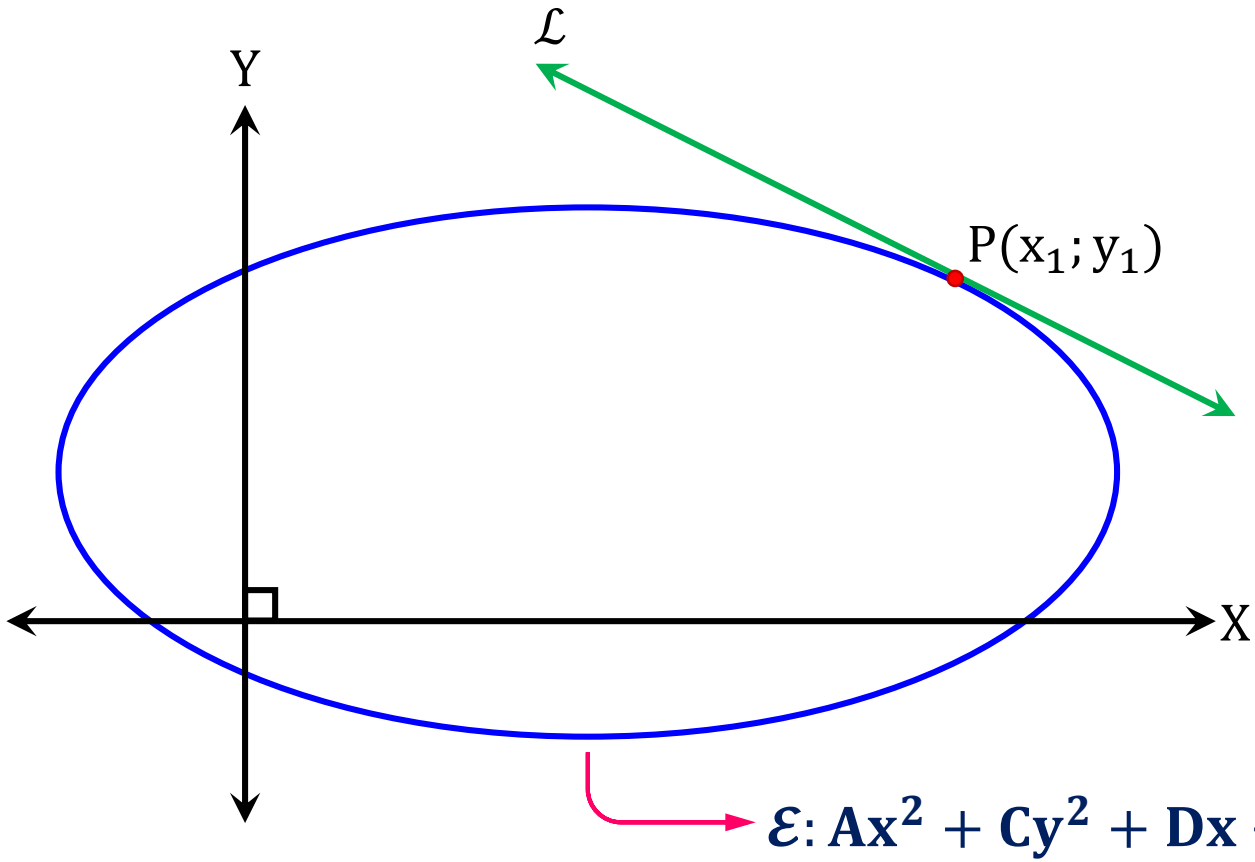
$$\mathcal{E}: Ax^2 + Cy^2 + Dx + Ey + F = 0$$

$AC > 0$

$A < C$

ELIPSE

d) Ecuación de la Recta Tangente a la Elipse:



Sea $P(x_1; y_1)$ el punto de tangencia de la recta $\mathcal{L}$ y la Elipse:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

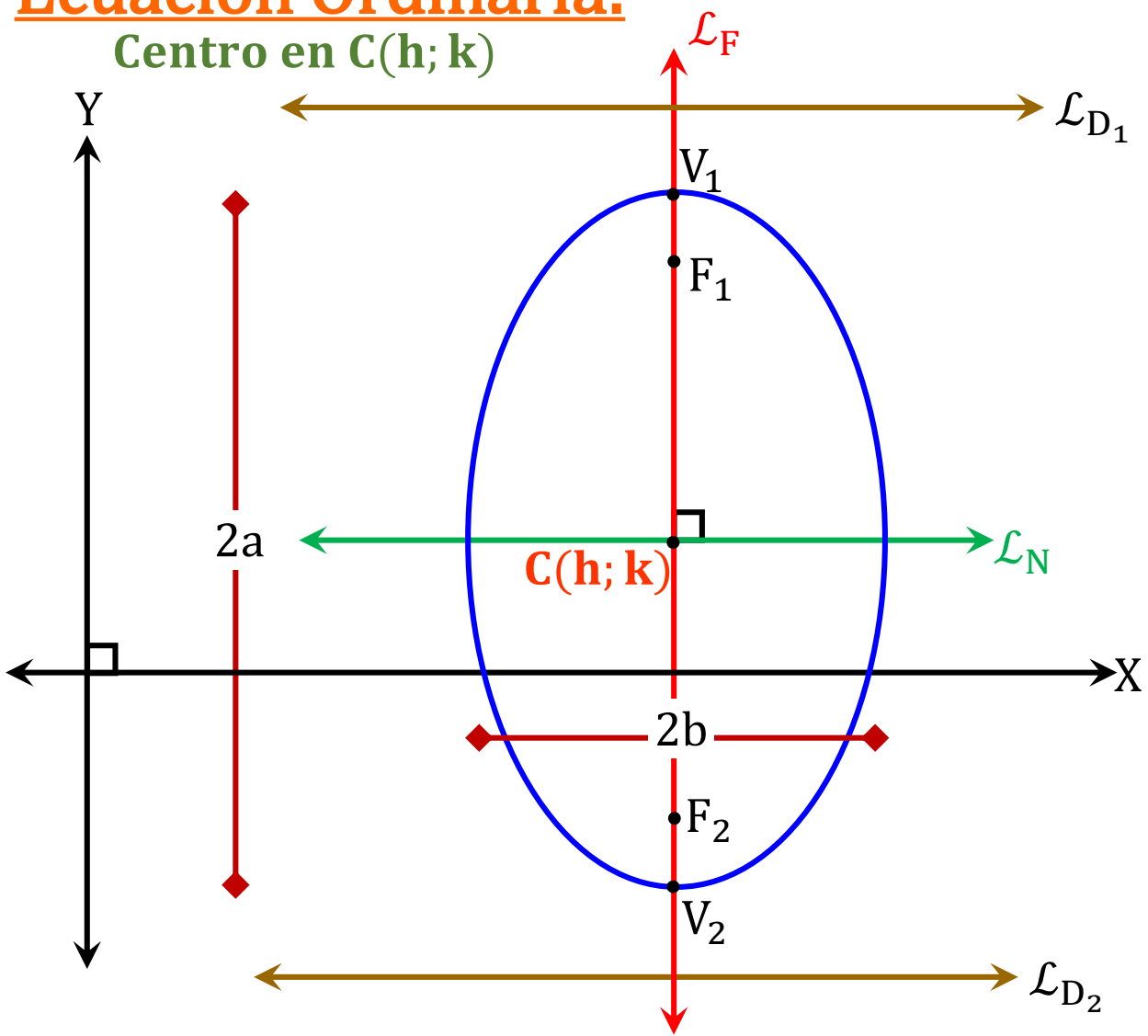
La ecuación de la recta $\overleftrightarrow{\mathcal{L}}$ es:

$$\mathcal{L}: \mathbf{Axx_1 + Cyy_1 + \frac{D}{2}(x + x_1) + \frac{E}{2}(y + y_1) + F = 0}$$

ELIPSE

4.2. Eje Focal paralelo al eje Y:

a) Ecuación Ordinaria:

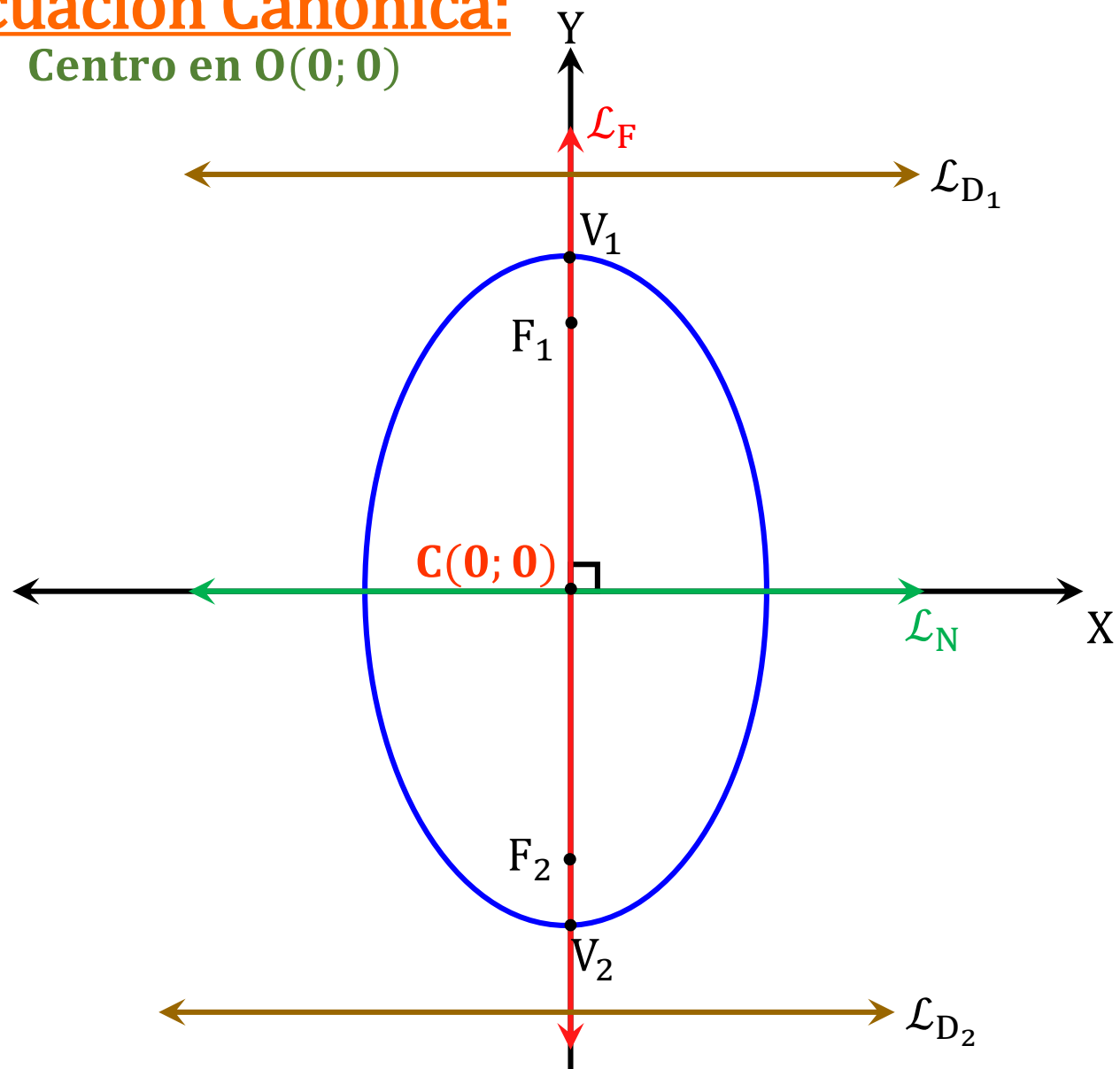


$$\mathcal{E}: \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

ELIPSE

b) Ecuación Canónica:

Centro en O(0; 0)



$$\mathcal{E}: \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

ELIPSE

c) Ecuación General:

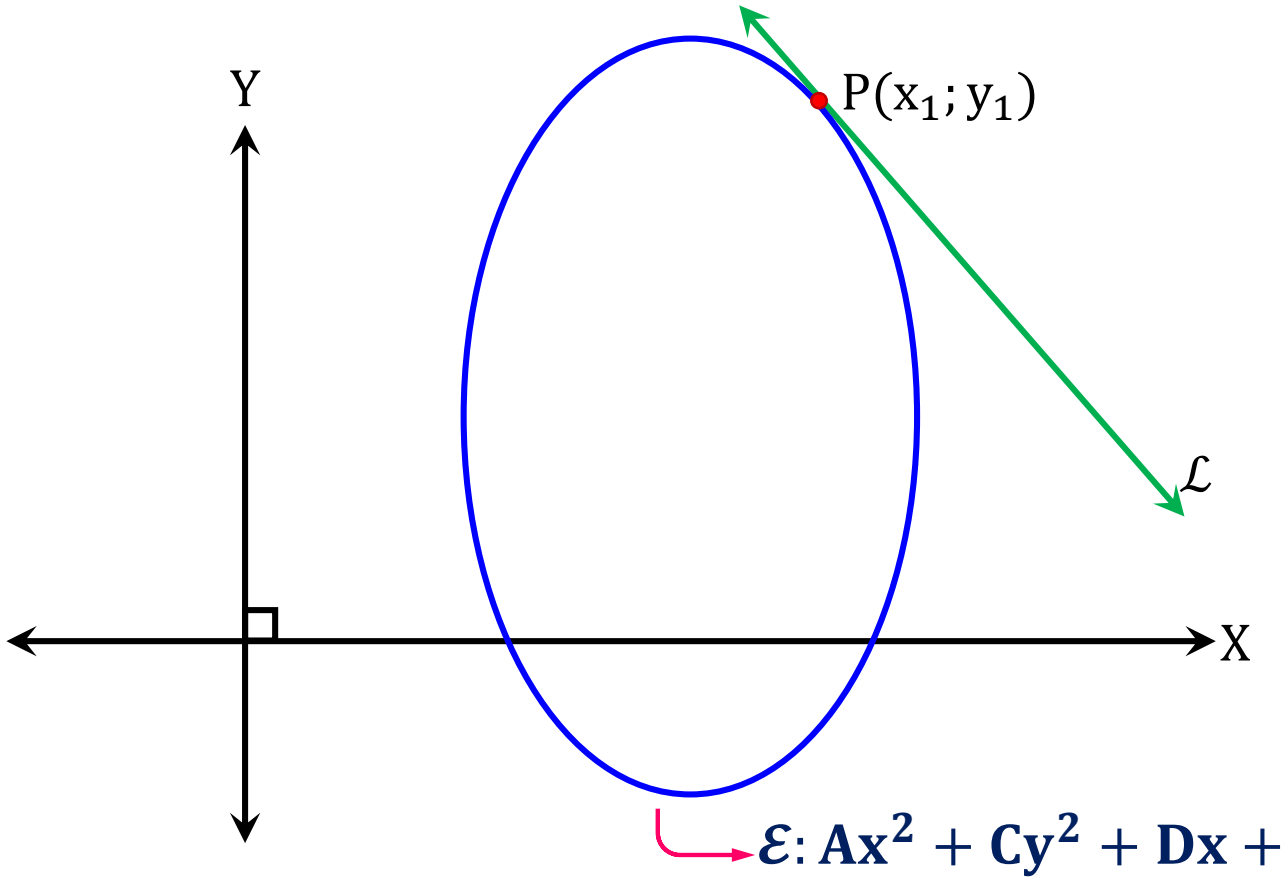
$$\mathcal{E}: Ax^2 + Cy^2 + Dx + Ey + F = 0$$

$AC > 0$

$A > C$

ELIPSE

d) Ecuación de la Recta Tangente a la Elipse:



Sea $P(x_1; y_1)$ el punto de tangencia de la recta $\mathcal{L}$ y la Elipse:

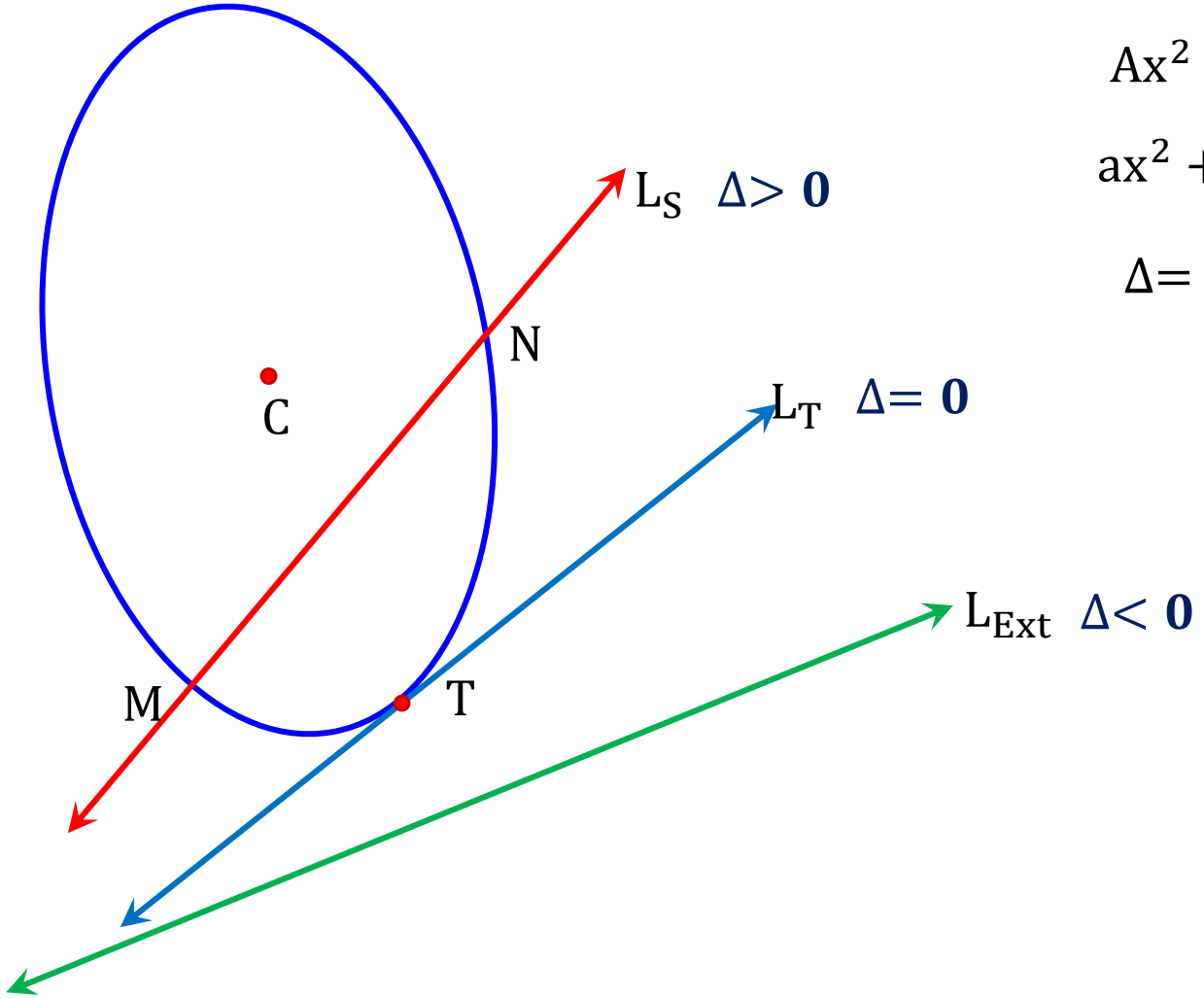
$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

La ecuación de la recta $\overleftrightarrow{\mathcal{L}}$ es:

$$\mathcal{L}: \mathbf{Axx_1 + Cyy_1 + \frac{D}{2}(x + x_1) + \frac{E}{2}(y + y_1) + F = 0}$$

ELIPSE

5. OBSERVACIÓN: Sea $\mathcal{E}: Ax^2 + Cy^2 + Dx + Ey + F = 0$ y la $\mathcal{L}: y = mx + b$

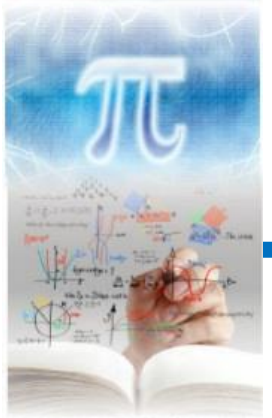


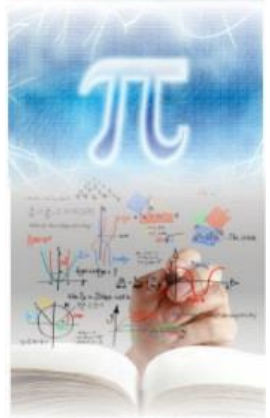
$$Ax^2 + C(mx + b)^2 + Dx + E(mx + b) + F = 0$$

$$ax^2 + bx + c = 0$$

$$\Delta = b^2 - 4ac$$

MOMENTO DE PRACTICAR



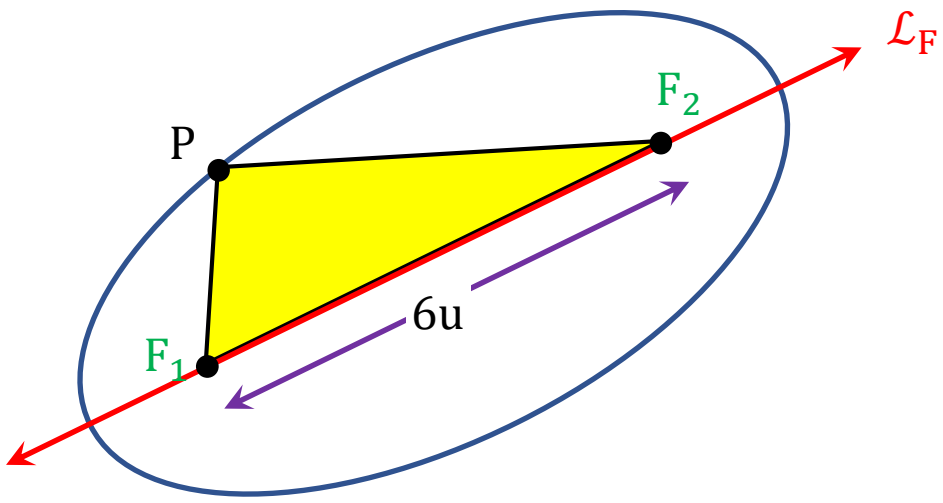


PROBLEMAS Y RESOLUCIÓN

ELIPSE

1. En una elipse de focos F_1 y F_2, siendo P un punto de la elipse, se tiene que $F_1F_2 = 6$ u y el perímetro del triángulo PF_1F_2 es 18 u. Luego la longitud de un lado recto es:

Resolución:



$$2p_{\Delta PF_1F_2} = 18u$$

$$\underbrace{PF_1 + PF_2}_{2a} + \underbrace{F_1F_2}_{6} = 18u$$

$$a = 6u$$

$$F_1F_2 = 2c$$

$$6u = 2c$$

$$c = 3u$$

$$a^2 = b^2 + c^2$$

$$6^2 = b^2 + 3^2$$

$$36 = b^2 + 9$$

$$b^2 = 27$$

$$LR = \frac{2b^2}{a}$$

$$LR = \frac{2(27)}{6}$$

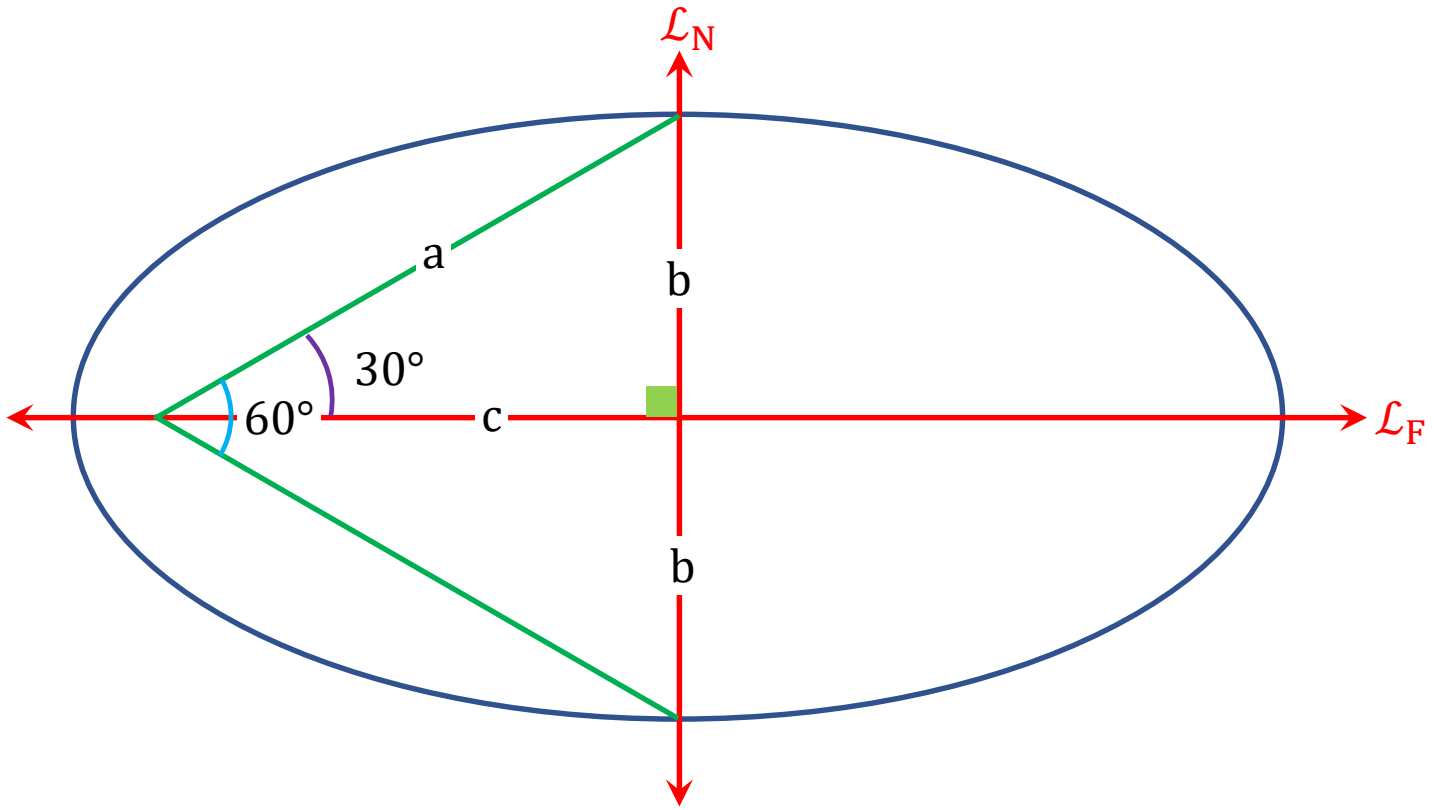
$$\therefore \mathbf{LR = 9}$$

CLAVE: D

ELIPSE

2. Determinar la excentricidad de la elipse, si: su eje menor se ve desde uno de los focos formando un ángulo de 60°.

Resolución:



$$e = \frac{c}{a}$$

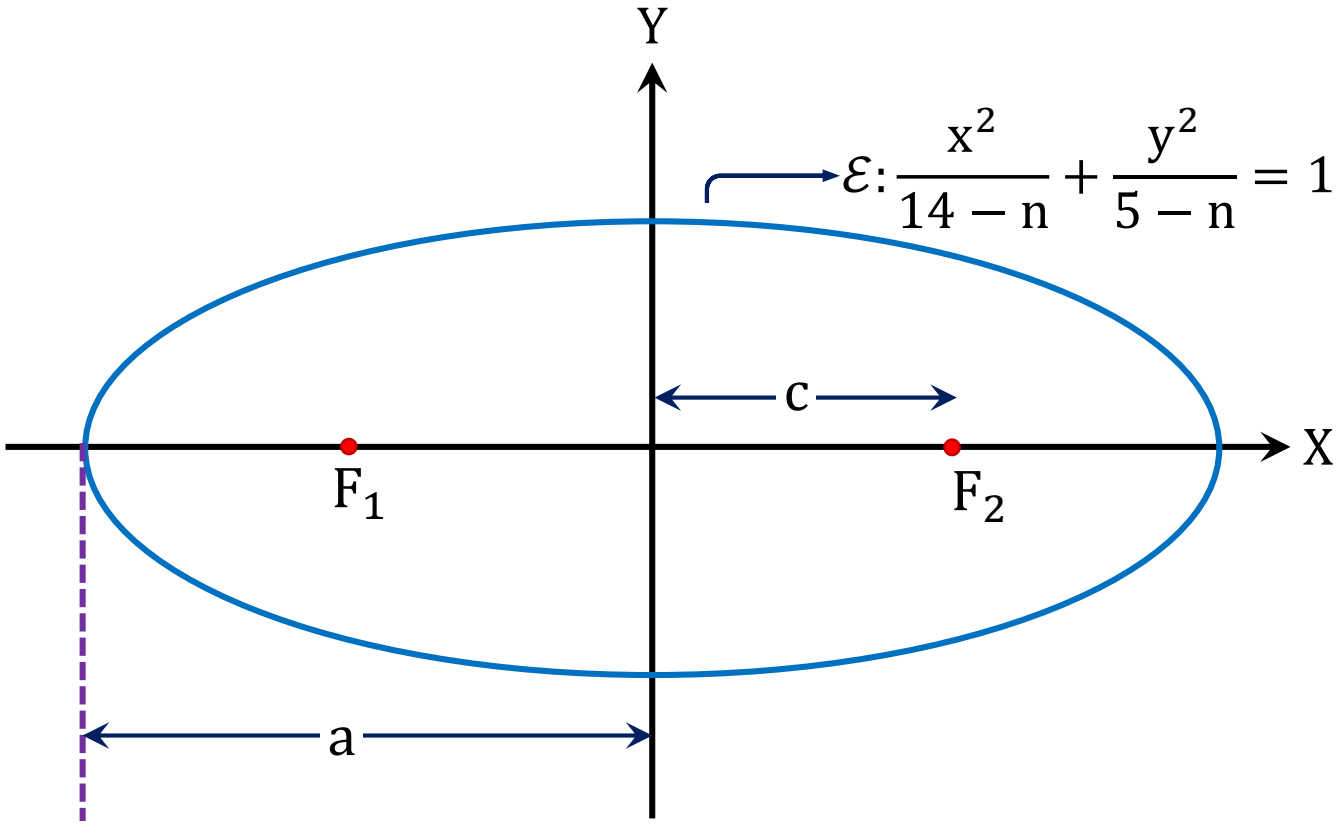
$$e = \cos 30°$$

$$\therefore \mathbf{e = \frac{\sqrt{3}}{2}}$$

CLAVE: D

ELIPSE

3. De la siguiente elipse de focos F_1 y F_2, calcule la longitud del eje menor si $a^2 = 4c$



Resolución:

$$\mathcal{E}: \frac{x^2}{\underbrace{14-n}_{a^2}} + \frac{y^2}{\underbrace{5-n}_{b^2}} = 1$$

$$a^2 = b^2 + c^2$$

$$14 - n = 5 - n + c^2$$

$$c = 3$$

$$\rightarrow a^2 = 4(3)$$

$$a^2 = 12$$

$$b = \sqrt{3}$$

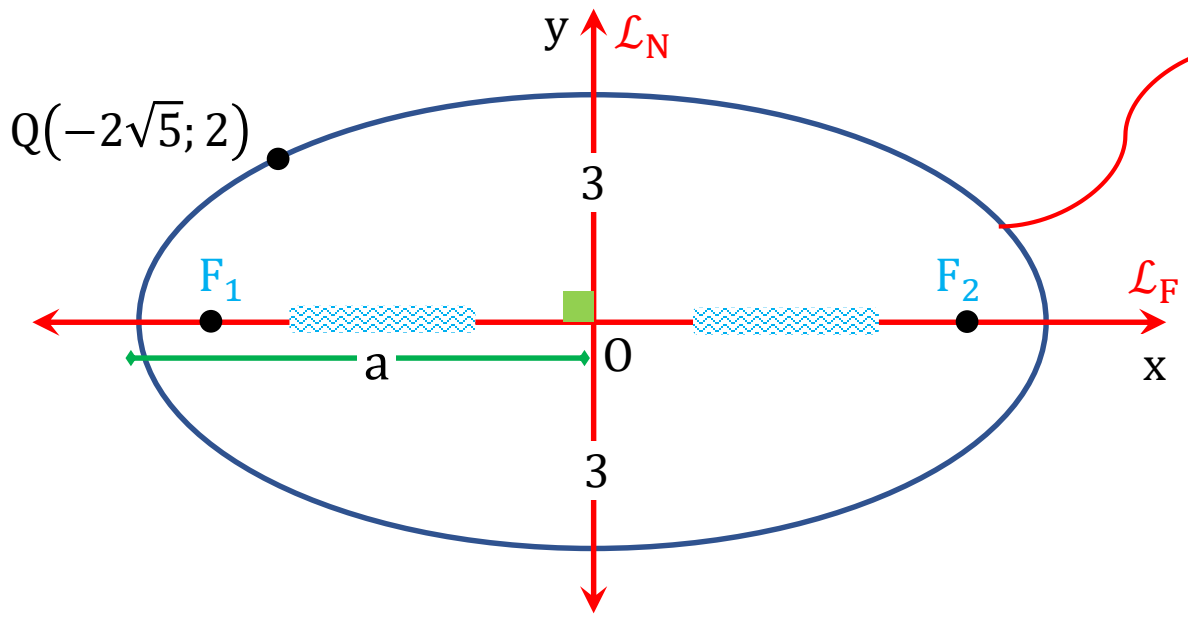
$$\therefore \mathbf{b = 2\sqrt{3}}$$

CLAVE: B

ELIPSE

4. Hallar la ecuación de la elipse cuyos focos están situados en el eje de abscisas y son simétricos con respecto al origen de coordenadas, si $Q(-2\sqrt{5};2)$ está en la elipse y su semieje menor $b = 3$

Resolución:



$$\mathcal{E}: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow \mathcal{E}: \frac{x^2}{a^2} + \frac{y^2}{(3)^2} = 1$$

$$Q(-2\sqrt{5};2) \in \mathcal{E}: \quad \frac{(-2\sqrt{5})^2}{a^2} + \frac{2^2}{9} = 1$$

$$\frac{20}{a^2} = 1 - \frac{4}{9}$$

$$\frac{20}{a^2} = \frac{5}{9}$$

$$a^2 = 36$$

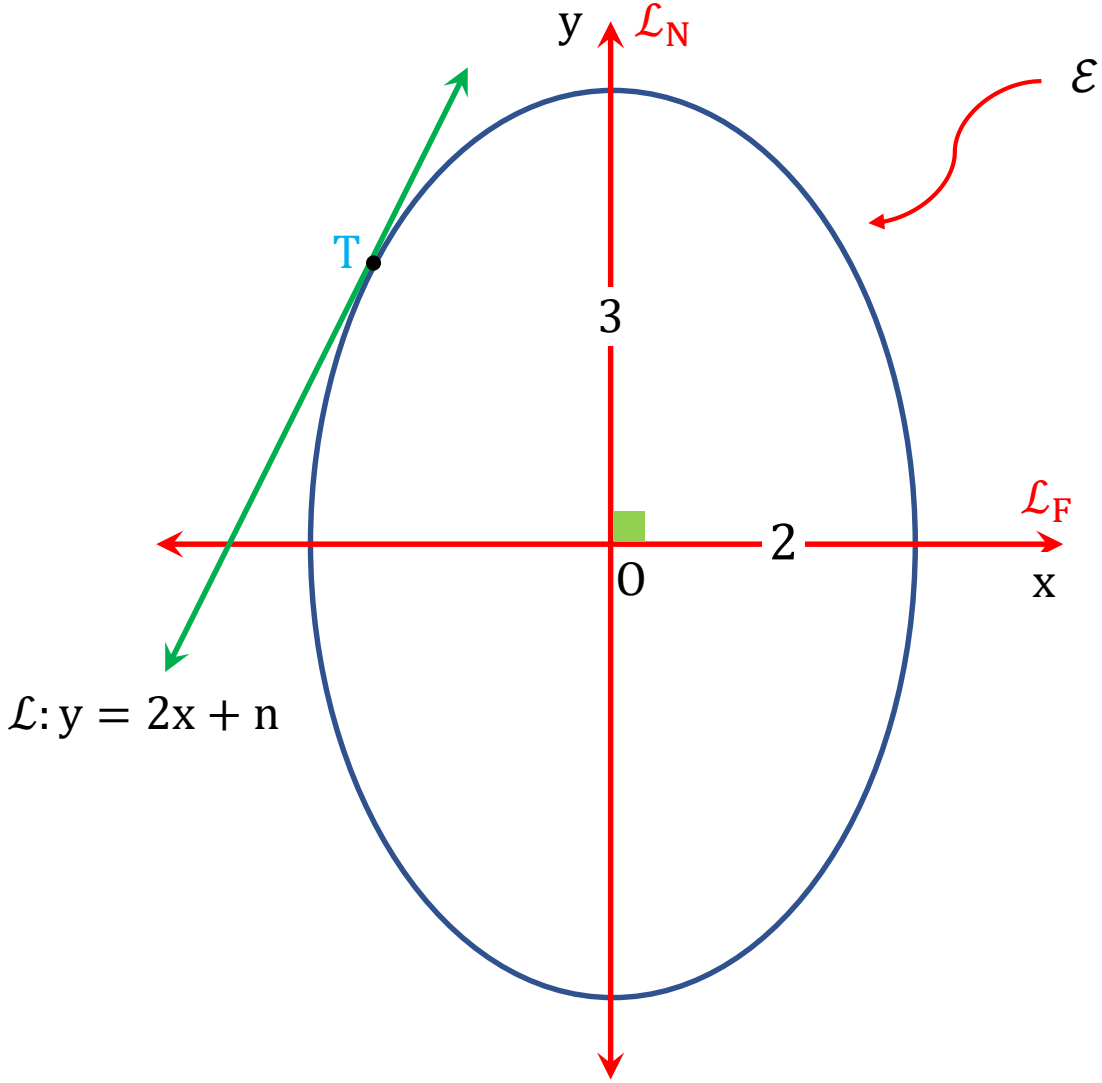
$$\therefore \mathcal{E}: \frac{x^2}{36} + \frac{y^2}{9} = 1$$

CLAVE: C

ELIPSE

5. Determinar n para que la recta $y = 2x + n$ sea tangente a la elipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Resolución:



$$\mathcal{E}: \frac{x^2}{2^2} + \frac{y^2}{3^2} = 1 \rightarrow \mathcal{E}: 9x^2 + 4y^2 = 36$$

$$9x^2 + 4(2x + n)^2 = 36$$

$$9x^2 + 4[4x^2 + 4xn + n^2] = 36$$

$$9x^2 + 16x^2 + 16xn + 4n^2 = 36$$

$$25x^2 + 16nx + 4n^2 - 36 = 0$$

$$\Delta = 0$$

$$(16n)^2 - 4.25.4(n^2 - 9) = 0$$

$$16n^2 - 25n^2 + 25.9 = 0$$

$$9n^2 = 25.9$$

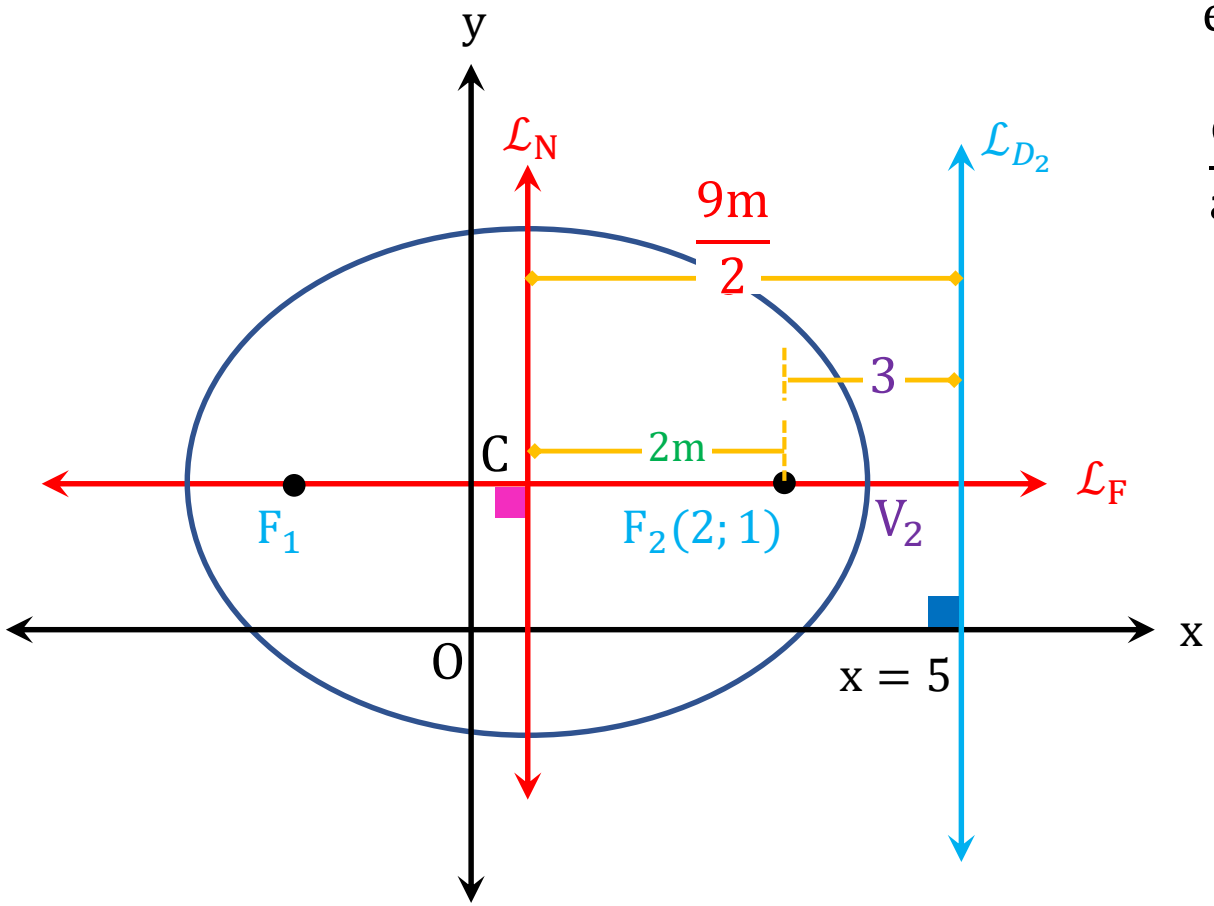
$$\therefore \mathbf{n = \pm 5}$$

CLAVE: E

ELIPSE

6. Hallar la ecuación de la elipse, si se conoce su excentricidad e = 2 / 3, su foco F(2; 1) y la ecuación de la directriz correspondiente $x - 5 = 0$

Resolución:



$$e = \frac{2}{3}$$

$$\frac{c}{a} = \frac{2}{3} \begin{cases} c = 2m \\ a = 3m \end{cases}$$

$$\rightarrow b = m\sqrt{5}$$

$$\frac{9m}{2} = 2m + 3$$

$$\frac{5m}{2} = 3$$

$$m = \frac{6}{5}$$

$$C(h;k) \rightarrow C(h;1)$$

$$h + 2m = 2$$

$$h = 2 - \frac{12}{5}$$

$$h = -\frac{2}{5}$$

$$\rightarrow C(-\frac{2}{5};1)$$

$$a = \frac{18}{5} \quad \wedge \quad b = \frac{6}{5}\sqrt{5}$$

ELIPSE

$$\mathcal{E}: \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\mathcal{E}: \frac{\left(x-\left(-\frac{2}{5}\right)\right)^2}{\left(\frac{18}{5}\right)^2} + \frac{(y-1)^2}{\left(\frac{6}{5}\sqrt{5}\right)^2} = 1$$

$$\mathcal{E}: \frac{\left(x+\frac{2}{5}\right)^2}{\frac{324}{25}} + \frac{(y-1)^2}{\frac{36}{5}} = 1$$

$$\mathcal{E}: \frac{25\left(x+\frac{2}{5}\right)^2}{324} + \frac{5(y-1)^2}{36} = 1$$

$$\mathcal{E}: \frac{25\left(x^2+\frac{4x}{5}+\frac{4}{25}\right)}{324} + \frac{5(y^2-2y+1)}{36} = 1$$

$$\mathcal{E}: 25\left(x^2+\frac{4x}{5}+\frac{4}{25}\right) + 45(y^2-2y+1) = 324$$

$$\mathcal{E}: 25x^2 + 20x + 4 + 45y^2 - 90y + 45 = 324$$

$$\mathcal{E}: 25x^2 + 45y^2 + 20x - 90y - 275 = 0$$

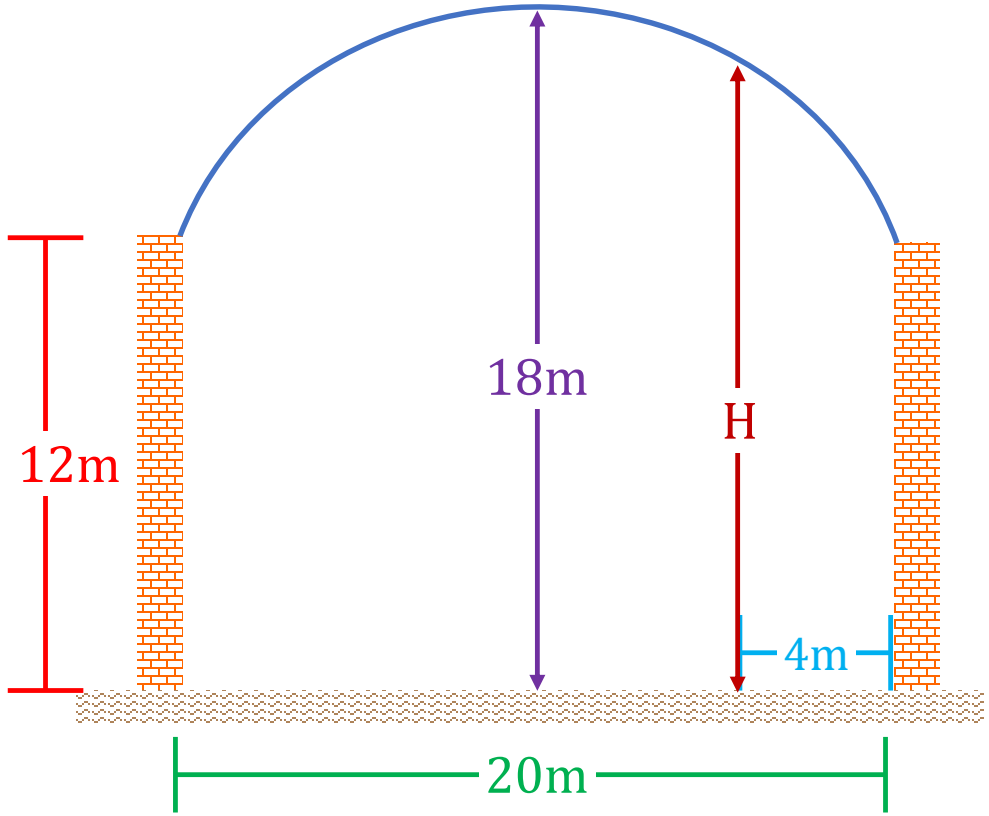
$$\therefore \mathcal{E}: \mathbf{5x^2 + 9y^2 + 4x - 18y - 55 = 0}$$

CLAVE: A

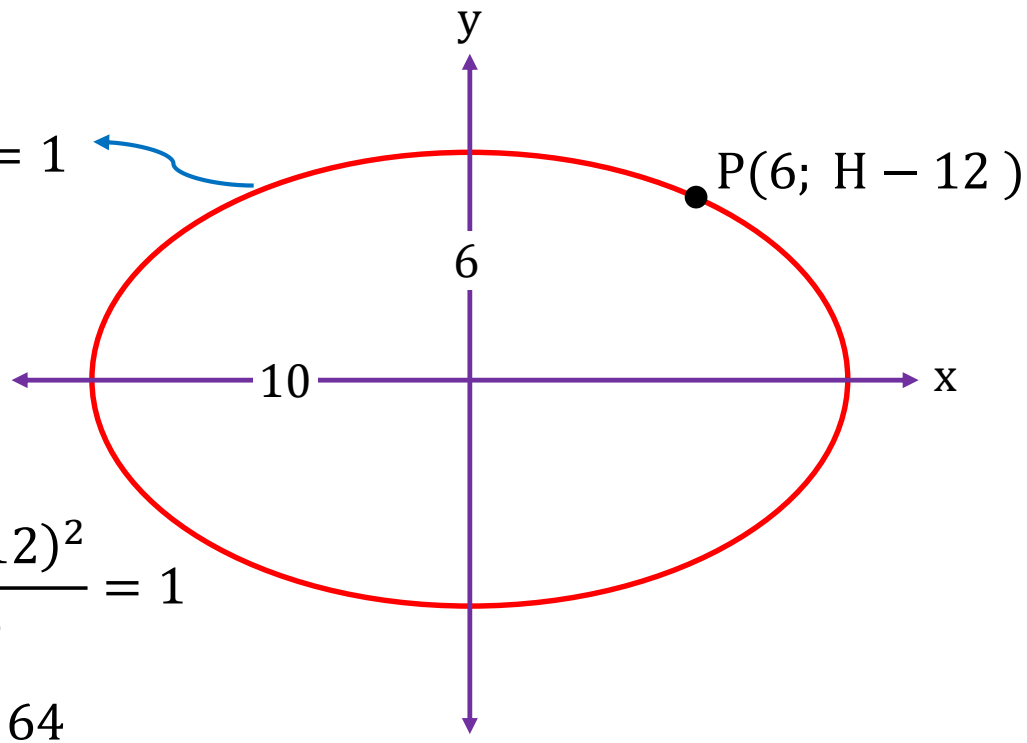
ELIPSE

7. El techo de una casa tiene la forma de una semi elipse. Si el ancho del mismo es de 20m y la altura de las paredes es 12 m. Determinar cuál es la altura del techo a 4 m de las paredes, si la altura total en el centro es de 18 m.

Resolución:



$$\mathcal{E}: \frac{x^2}{10^2} + \frac{y^2}{6^2} = 1$$



$P \in \mathcal{E}$:

$$\frac{(6)^2}{100} + \frac{(H-12)^2}{36} = 1$$

$$\frac{(H-12)^2}{36} = \frac{64}{100}$$

$$H - 12 = \frac{6.8}{10}$$

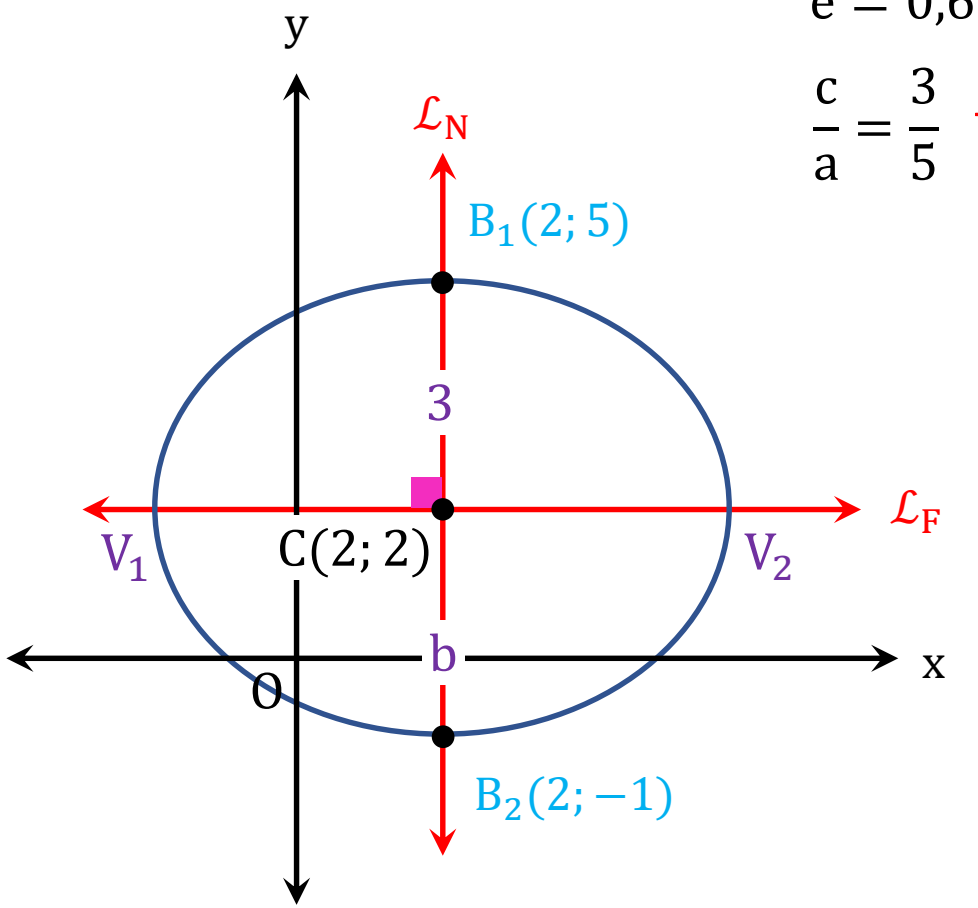
$$\therefore \mathbf{H = 16,8}$$

CLAVE: D

ELIPSE

8. Los extremos del eje menor de una elipse son los puntos $(2;\ -1)$ y $(2;\ 5)$. Hallar la ecuación de la elipse, cuya excentricidad es 0,6.

Resolución:



$$e = 0{,}6$$

$$\frac{c}{a} = \frac{3}{5} \begin{cases} c = 3q \\ a = 5q \end{cases} \rightarrow b = 4q \rightarrow 3 = 4q \rightarrow q = \frac{3}{4}$$

$$\mathcal{E}: \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \rightarrow \mathcal{E}: \frac{(x-2)^2}{\left(\frac{15}{4}\right)^2} + \frac{(y-2)^2}{(3)^2} = 1$$

$$\mathcal{E}: \frac{16(x^2-4x+4)}{225} + \frac{(y^2-4y+4)}{9} = 1$$

$$\mathcal{E}: 16(x^2-4x+4) + 25(y^2-4y+4) = 225$$

$$\mathcal{E}: 16x^2 - 64x + 64 + 25y^2 - 100y + 100 = 225$$

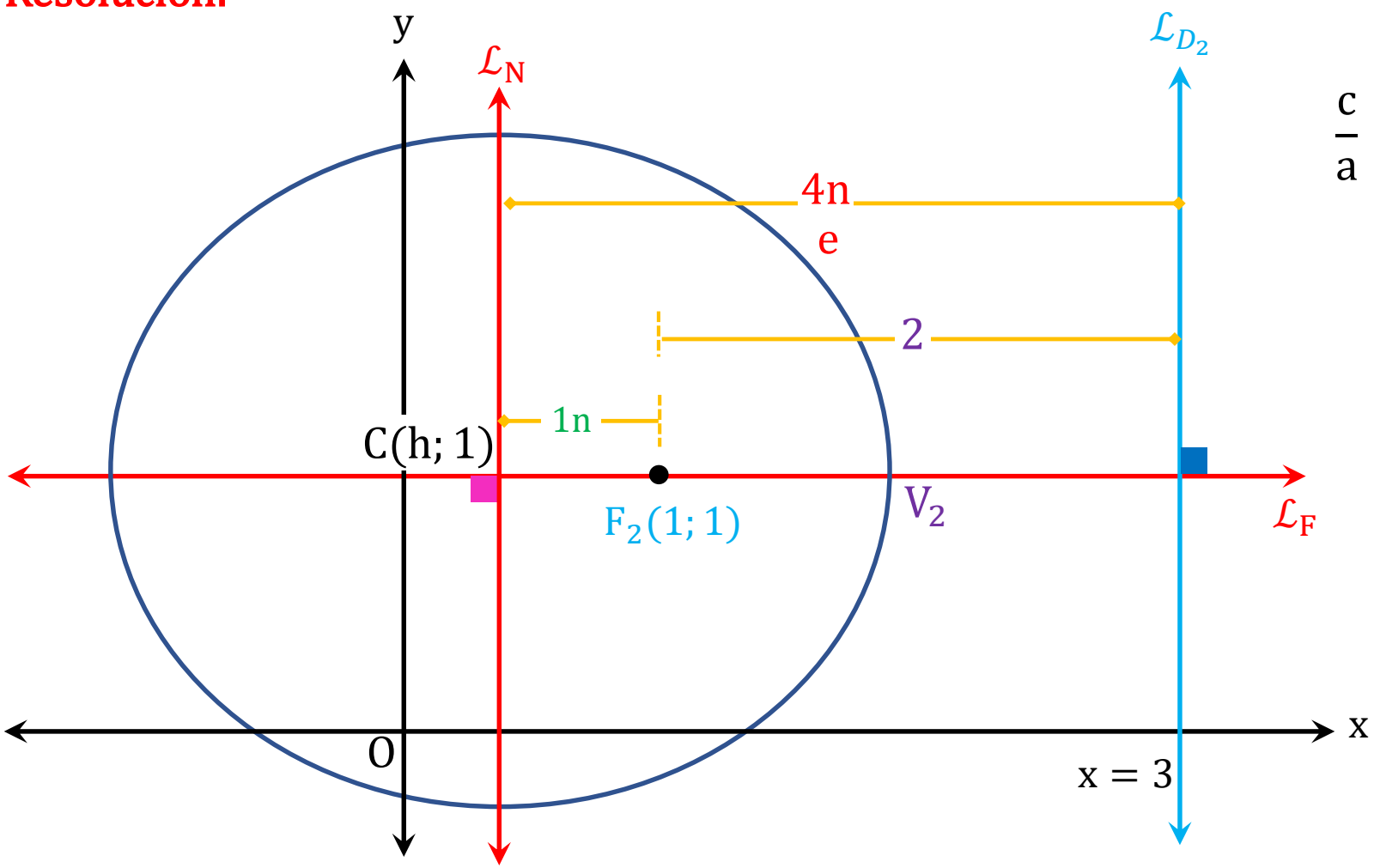
$$\therefore \mathcal{E}: \mathbf{16x^2 + 25y^2 - 64x - 100y - 61 = 0}$$

CLAVE: D

ELIPSE

9. Si el foco de una elipse es el punto (1; 1) ; su directriz correspondiente es la recta x = 3 y su excentricidad es 0,5 ; entonces determine la ecuación de dicha elipse.

Resolución:



$e = 0{,}5$

$$\frac{c}{a} = \frac{1}{2} \begin{cases} c = 1n \\ a = 2n \end{cases}$$

$$\rightarrow b = n\sqrt{3}$$

$$4n = n + 2$$

$$3n = 2$$

$$n = \frac{2}{3}$$

$C(h; k) \rightarrow C(h; 1)$

$$h + n = 1$$

$$h = 1 - \frac{2}{3}$$

$$h = \frac{1}{3}$$

$$\rightarrow C\left(\frac{1}{3}; 1\right)$$

$$a = \frac{4}{3} \quad \wedge \quad b = \frac{2}{3}\sqrt{3}$$

ELIPSE

$$\mathcal{E}: \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\mathcal{E}: \frac{\left(x-\frac{1}{3}\right)^2}{\left(\frac{4}{3}\right)^2} + \frac{(y-1)^2}{\left(\frac{2}{3}\sqrt{3}\right)^2} = 1$$

$$\mathcal{E}: \frac{\left(x-\frac{1}{3}\right)^2}{\frac{16}{9}} + \frac{(y-1)^2}{\frac{4}{3}} = 1$$

$$\mathcal{E}: \frac{9\left(x-\frac{1}{3}\right)^2}{16} + \frac{3(y-1)^2}{4} = 1$$

$$\mathcal{E}: \frac{9\left(x^2-\frac{2x}{3}+\frac{1}{9}\right)}{16} + \frac{3(y^2-2y+1)}{4} = 1$$

$$\mathcal{E}: 9\left(x^2+\frac{2x}{3}+\frac{1}{9}\right) + 12(y^2-2y+1) = 16$$

$$\mathcal{E}: 9x^2+6x+1+12y^2-24y+12 = 16$$

$$\mathcal{E}: 9x^2+12y^2+6x-24y-3 = 0$$

$$\therefore \mathcal{E}: \mathbf{3x^2+4y^2+2x-8y-1=0}$$

CLAVE: B

ELIPSE

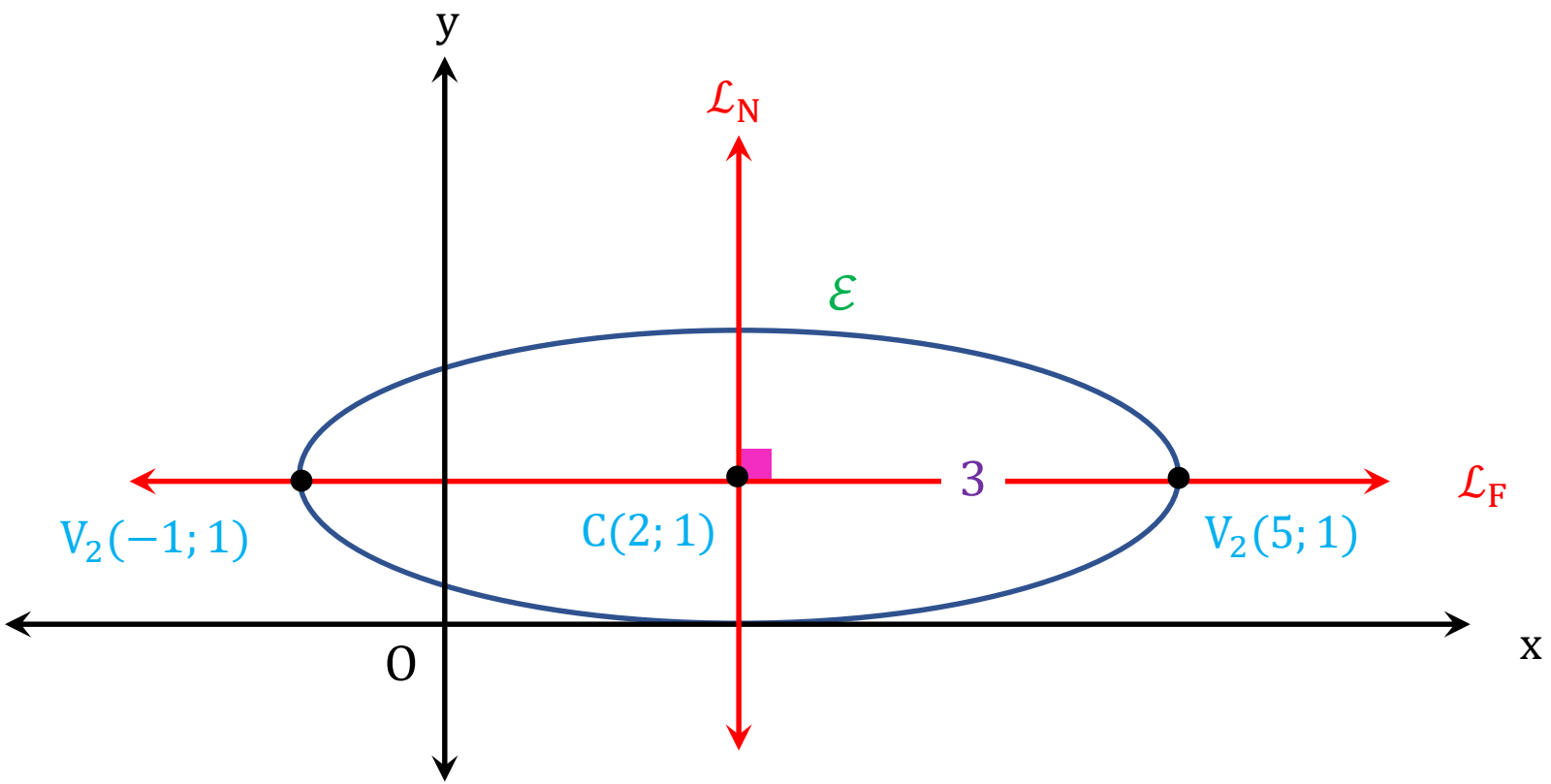
10. Determine la ecuación de la recta tangente trazada desde un vértice $(x < 0)$ de la elipse de ecuación:

$$\mathcal{E}: \frac{(x-2)^2}{9} + \frac{(y-1)^2}{1} = 1$$

A la parábola: $3y^2 - 5x - 8 = 0$

Resolución:

$C(2;1)$ y $a = 3; b = 1$



$$\mathcal{P}: 3y^2 - 5x - 8 = 0$$

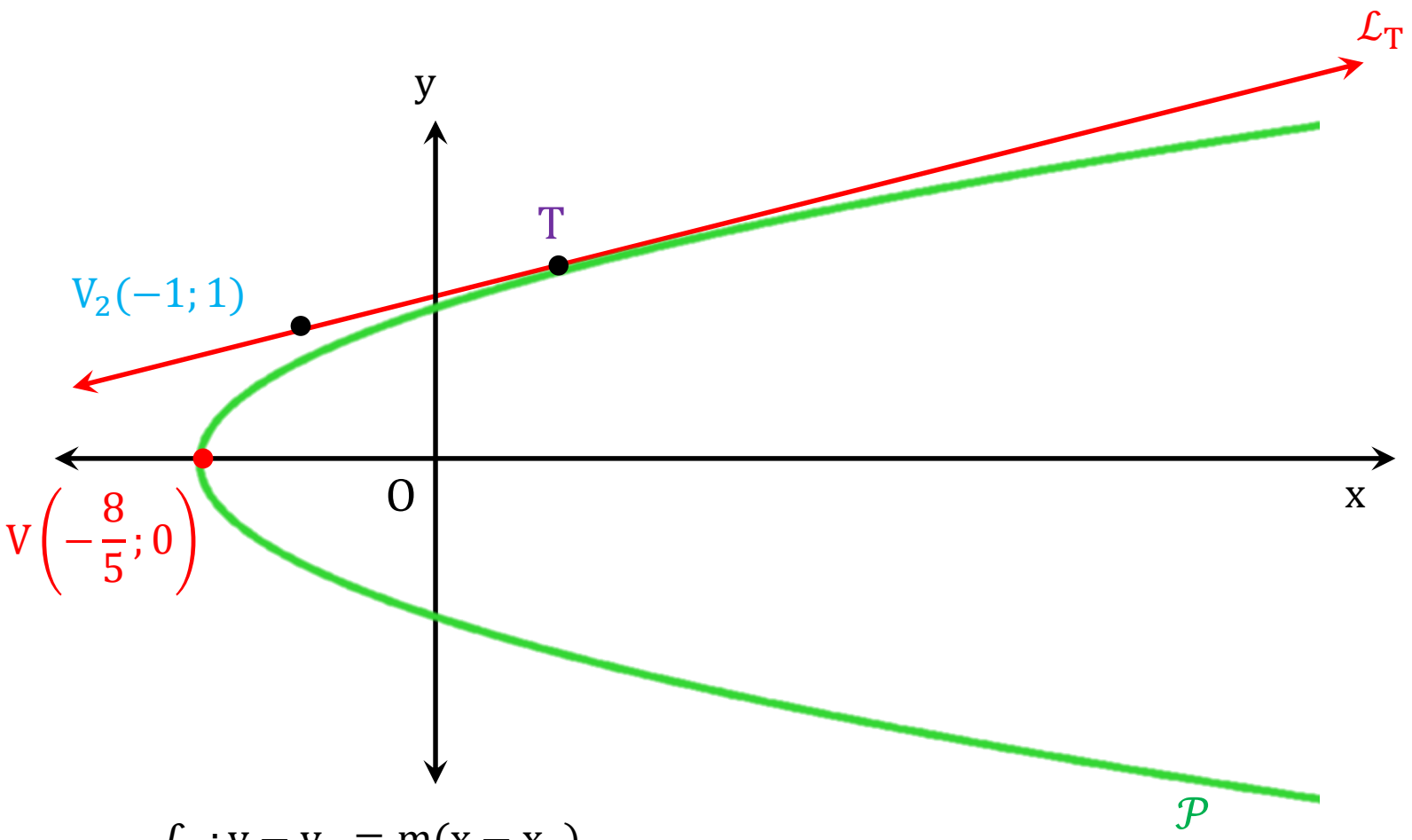
$$\mathcal{P}: 3y^2 = 5x + 8$$

$$\mathcal{P}: y^2 = \frac{1}{3}(5x + 8)$$

$$\mathcal{P}: y^2 = \frac{5}{3}\left(x + \frac{8}{5}\right)$$

$$V\left(-\frac{8}{5}; 0\right) \text{ y } 4p = \frac{5}{3} \rightarrow p = \frac{5}{12}$$

ELIPSE



$$\mathcal{P}: 5x = 3y^2 - 8$$

$$5\left(\frac{y-(m+1)}{m}\right) = 3y^2 - 8$$

$$5y - 5(m+1) = 3my^2 - 8m$$

$$3my^2 - 5y + 5 - 3m = 0$$

$$\Delta = 0$$

$$(-5)^2 - 4(3m)(5-3m) = 0$$

$$36m^2 - 60m + 25 = 0$$

$$(6m-5)^2 = 0$$

$$\mathbf{m = \frac{5}{6}}$$

$$\mathcal{L}_T: y - y_0 = m(x - x_0)$$

$$y - 1 = m(x+1)$$

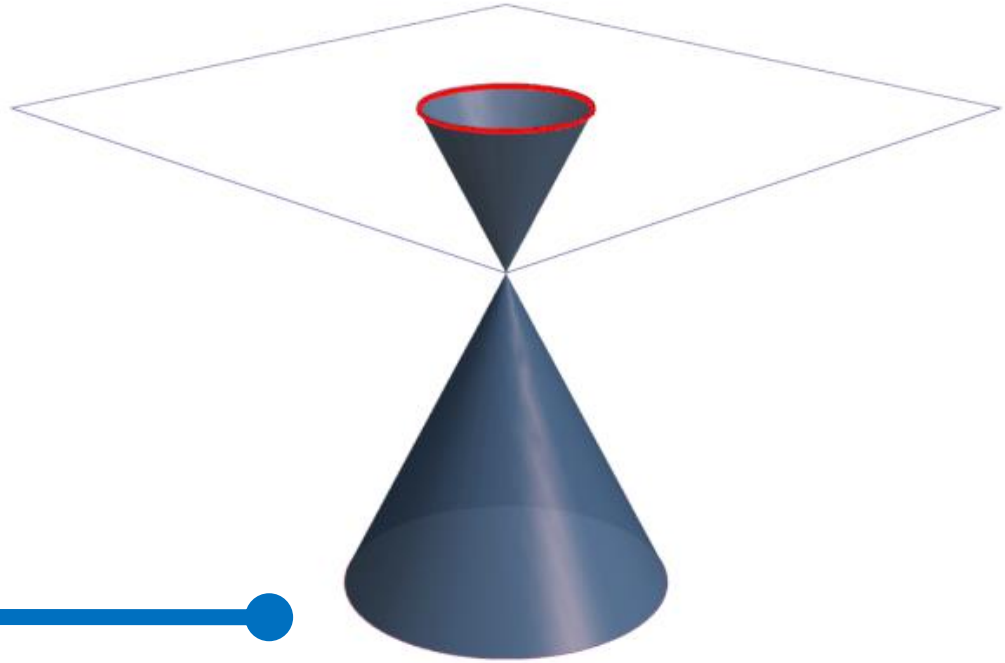
$$y - 1 = mx + m$$

$$x = \frac{y-(m+1)}{m}$$

$$\mathbf{\mathcal{L}_T: y - 1 = \frac{5}{6}(x+1)}$$

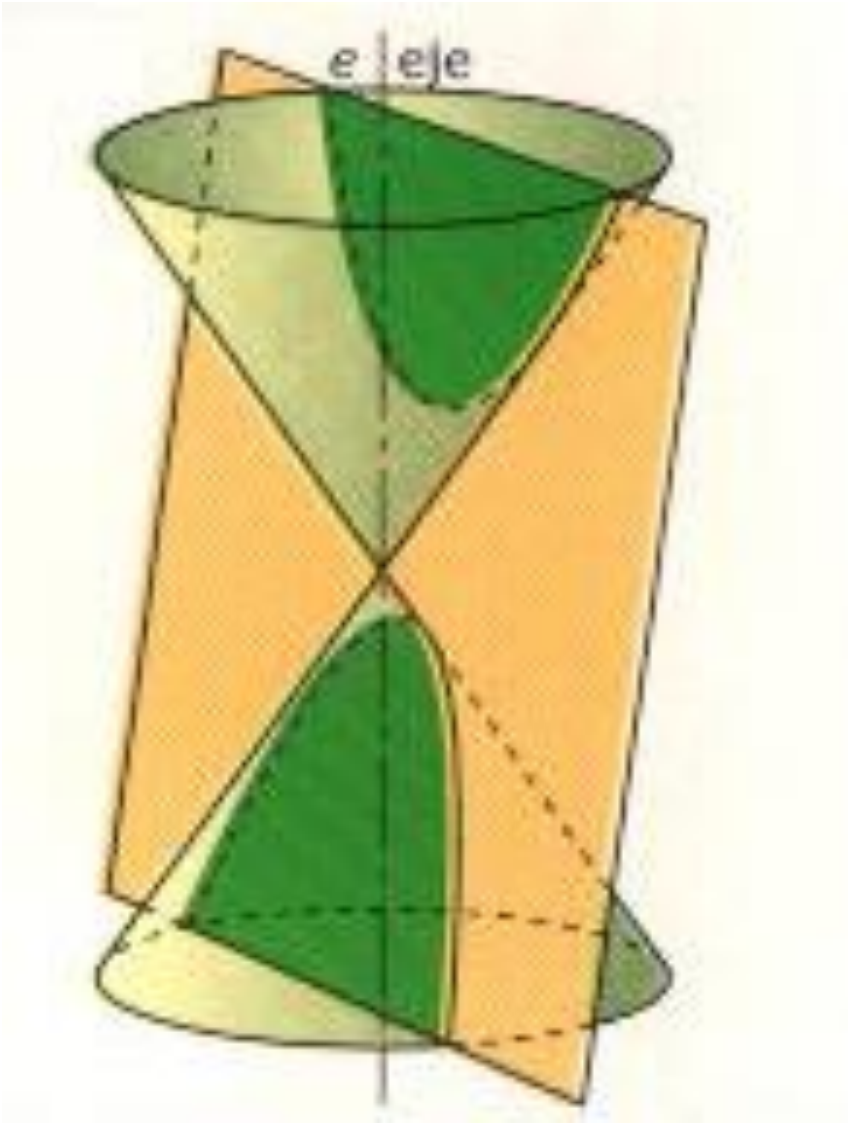
CLAVE: B

CÓNICAS



HIPERBOLA

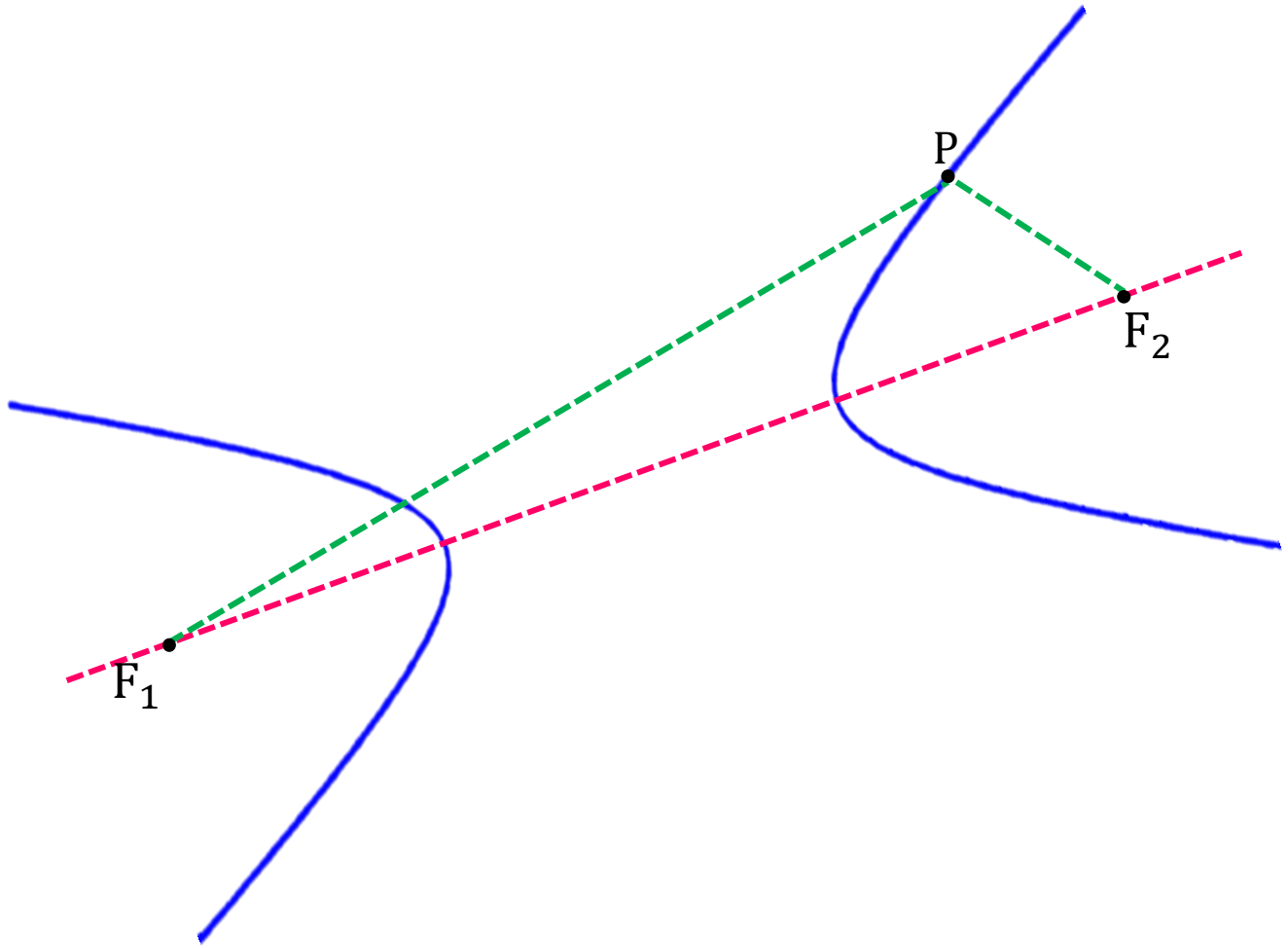
HIPÉRBOLA



HIPÉRBOLA

1. DEFINICIÓN:

Es aquel lugar geométrico de todos los puntos que pertenecen a un mismo plano, cuya diferencia de distancias (en valor absoluto) a dos puntos fijos del plano llamados focos, es constante.

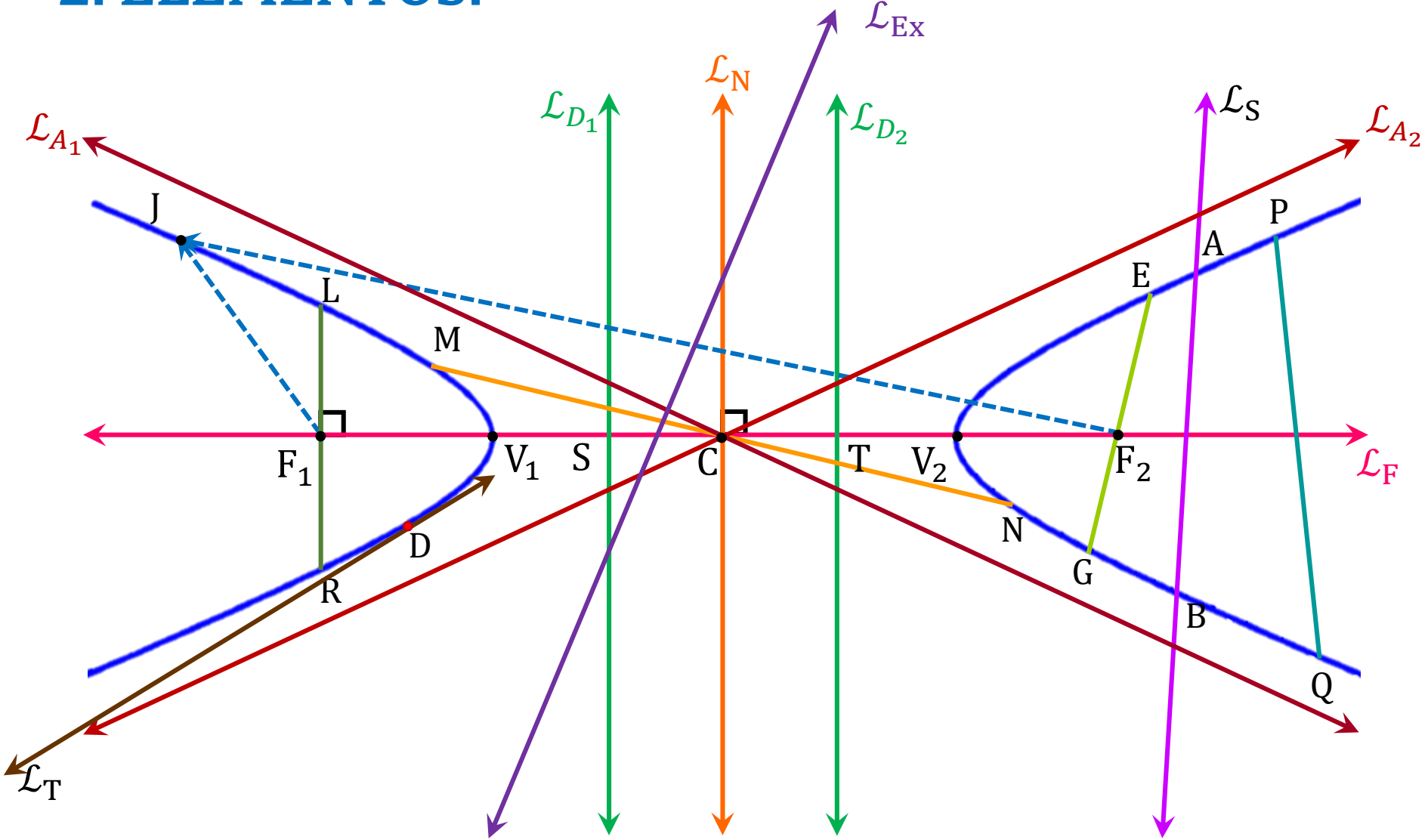


$$|d\overline{PF_1} - d\overline{PF_2}| = cte.$$

$$\overline{PF_1} - \overline{PF_2} = 2a$$

HIPÉRBOLA

2. ELEMENTOS:



$\mathcal{L}_F$: Eje Focal

$\mathcal{L}_N$: Eje Normal

C: Centro

$\mathcal{L}_{A_1}$ y $\mathcal{L}_{A_2}$:Asíntotas

F_1 y F_2: Focos

V_1 y V_2: Vértices

$\overline{PQ}$: Cuerda

$\overline{EG}$: Cuerda Focal

$\overline{MN}$: Diámetro

$\overline{LR}$: Lado Recto

F_1J y F_2J: Radios Focales

$\mathcal{L}_T$: Recta Tangente

$\mathcal{L}_S$: Recta Secante

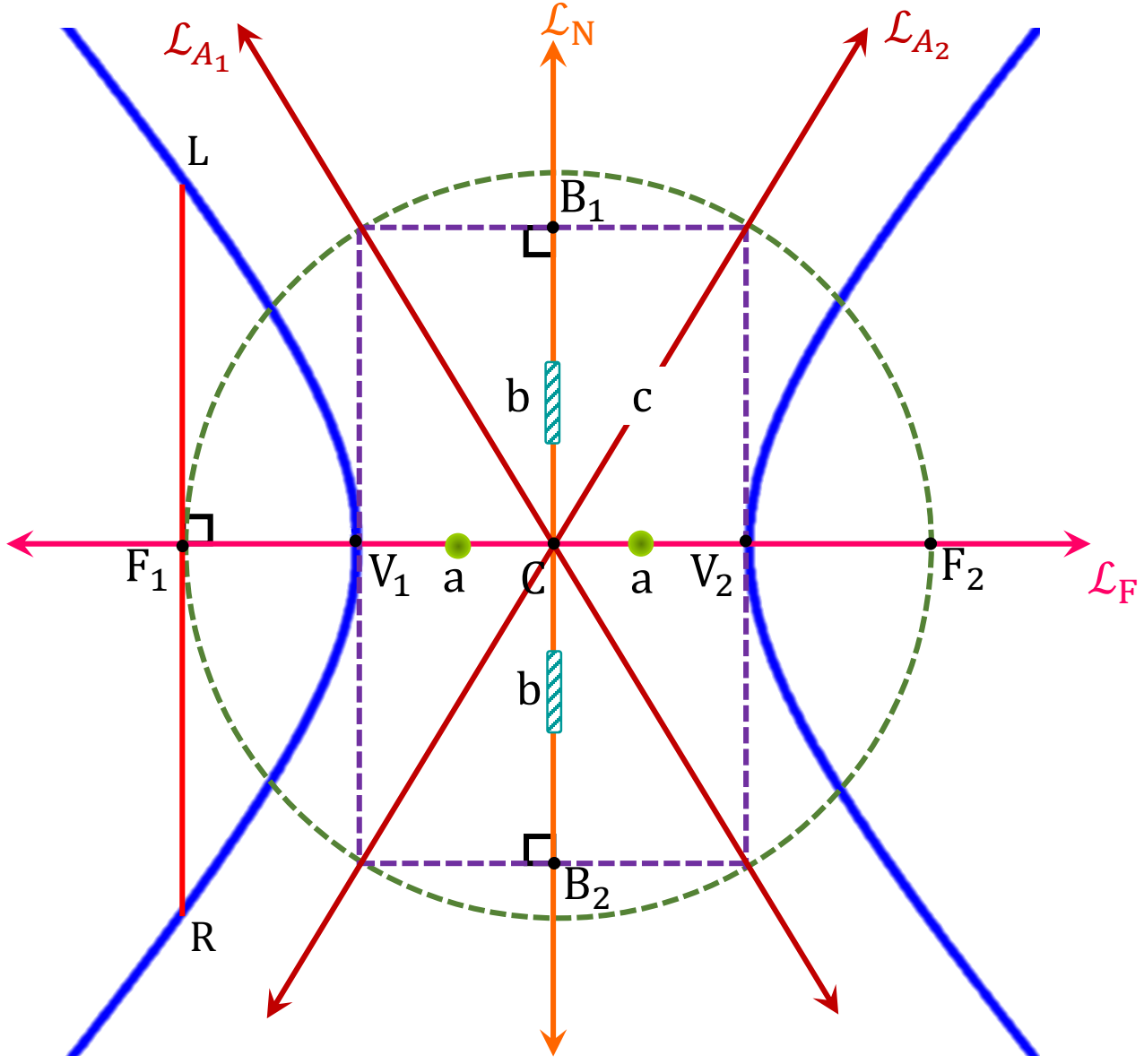
$\mathcal{L}_{Ex}$: Recta Exterior

$\mathcal{L}_{D_1}$ y $\mathcal{L}_{D_2}$: Recta Directriz

$\overline{ST}$: Distancia entre Directrices

HIPÉRBOLA

3. RELACIONES FUNDAMENTALES:



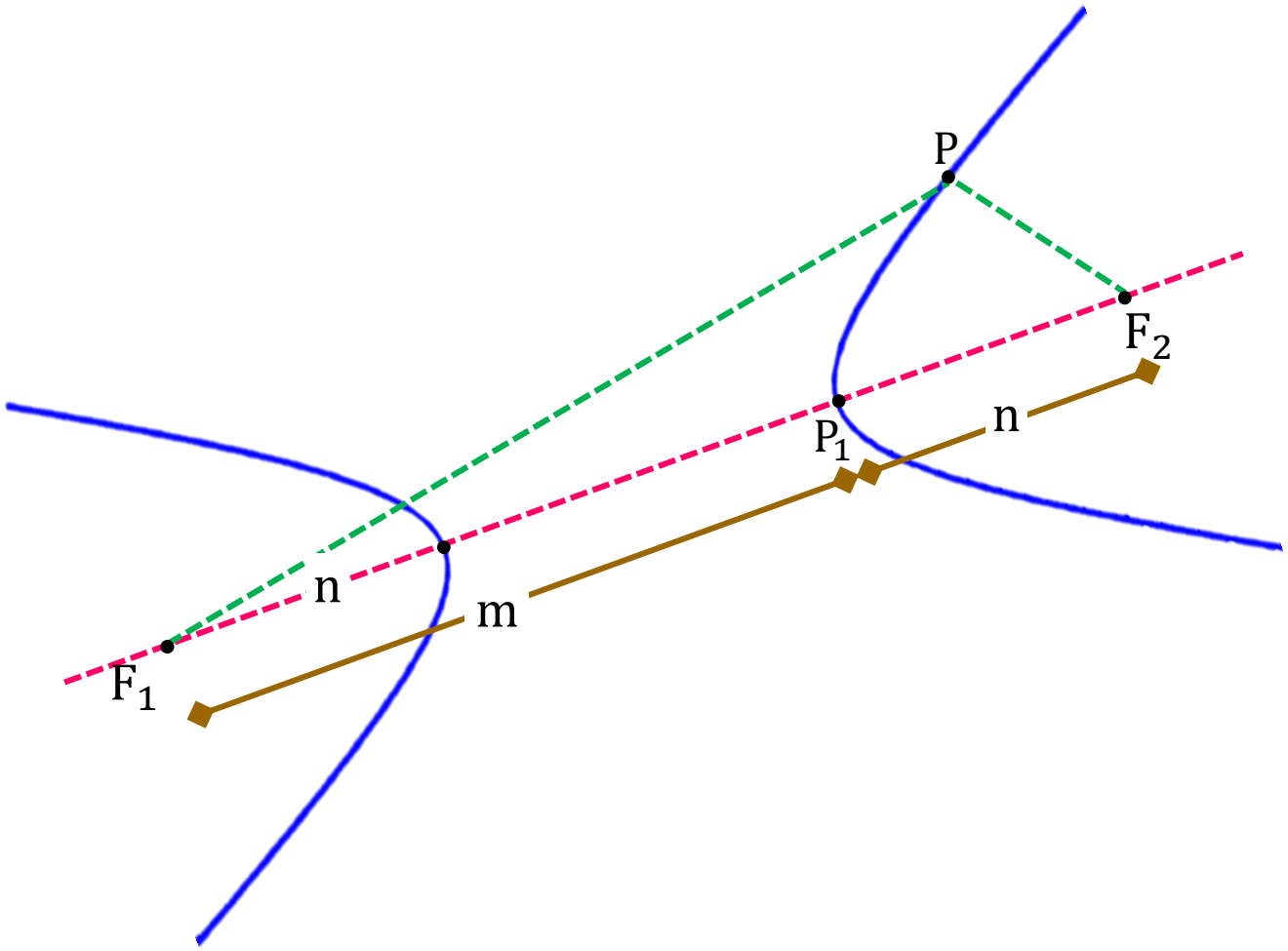
$$c^2 = a^2 + b^2$$

$c > a \wedge c > b$

$a > b\,,\, a = b \wedge a < b$

- $\overline{V_1\,V_2}$: Eje transverso $= 2a$
- $\overline{F_1\,F_2}$: Segmento focal $= 2c$
- $\overline{B_1\,B_2}$: Eje conjugado $= 2b$
- $\overline{LR} = \dfrac{2a^2}{b}$
- $e = \dfrac{c}{a}\,,\, e > 1$
- $\overline{ST} = \dfrac{2a}{e} = \dfrac{2a^2}{c}$

HIPÉRBOLA



$$|d\overline{PF_1} - d\overline{PF_2}| = cte.$$

$$\overline{PF_1} - \overline{PF_2} = 2a$$

$$m - n = 2a$$

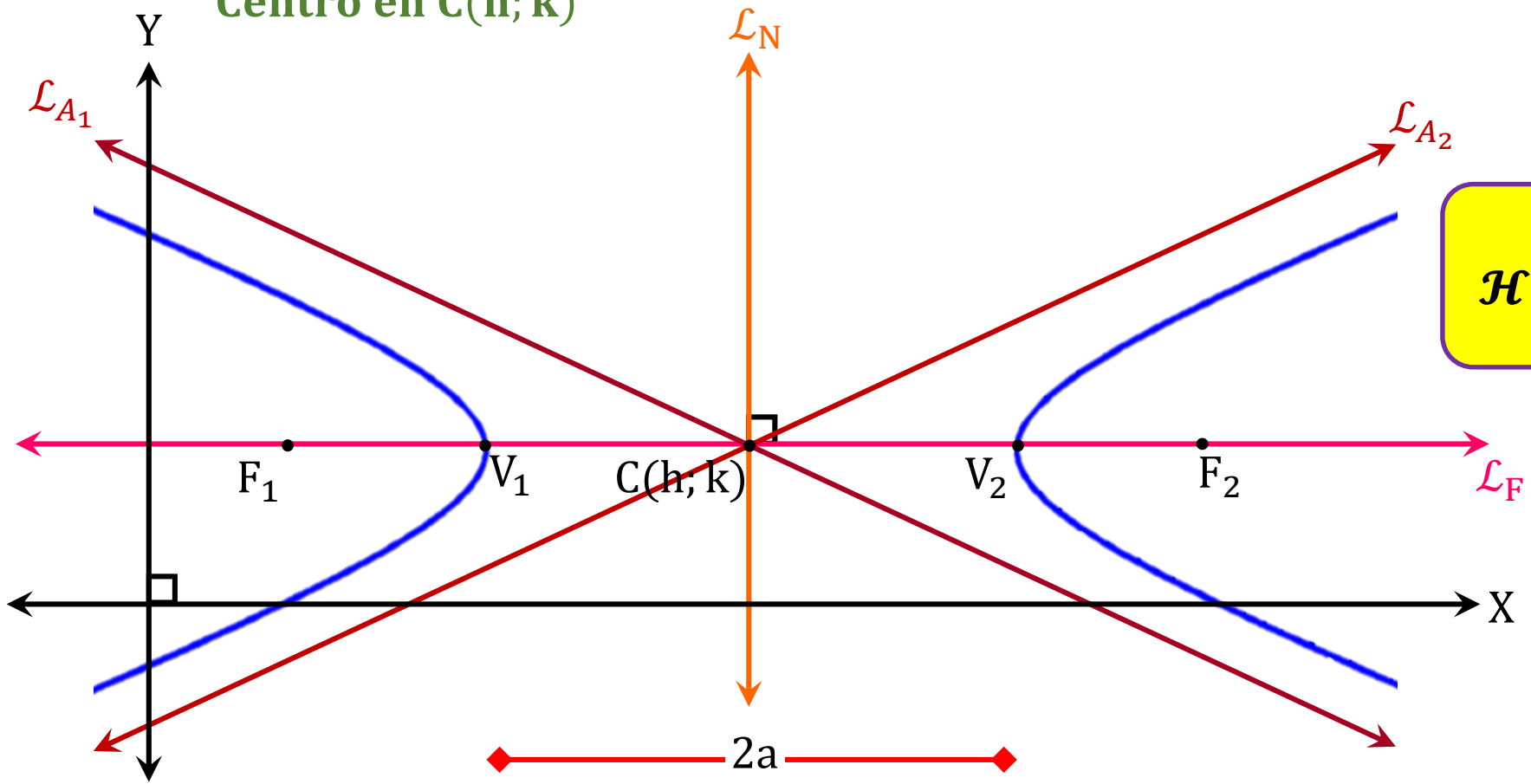
HIPÉRBOLA

4. ECUACIÓN DE LA HIPÉRBOLA:

4.1. Eje Focal paralelo al eje X:

a) Ecuación Ordinaria:

Centro en C(h; k)

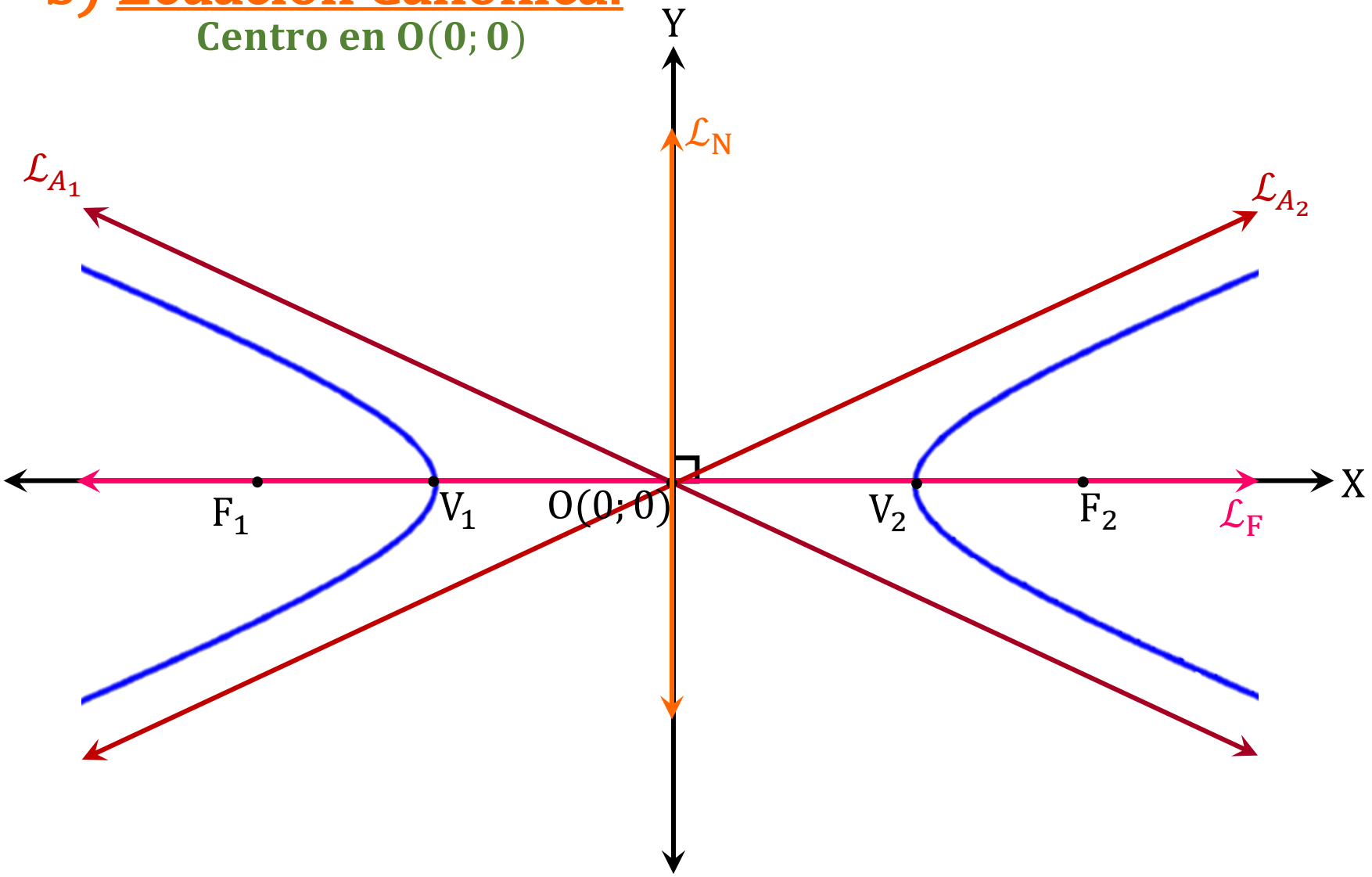


$$\mathcal{H}: \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

HIPÉRBOLA

b) Ecuación Canónica:

Centro en O(0; 0)



$$\mathcal{H}: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

HIPÉRBOLA

c) Ecuación General:

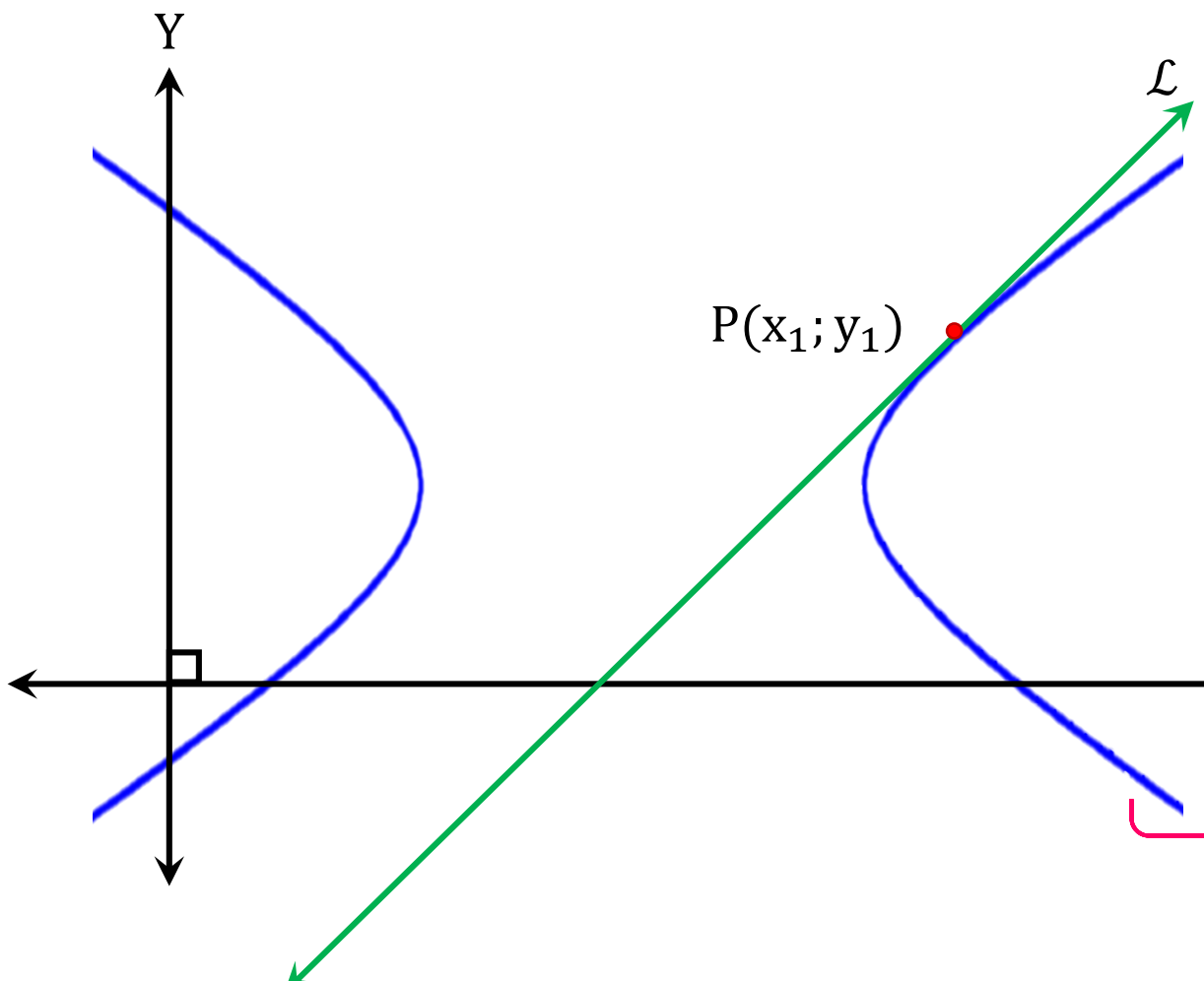
$$\mathcal{H}: Ax^2 + Cy^2 + Dx + Ey + F = 0$$

$AC < 0$

$A > 0 \ \wedge C < 0$*

HIPÉRBOLA

d) Ecuación de la Recta Tangente a la Hipérbola:



Sea $P(x_1; y_1)$ el punto de tangencia de la recta $\mathcal{L}$ y la Hipérbola:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

La ecuación de la recta $\overleftrightarrow{L}$ es:

$$\mathcal{L}: \mathbf{Axx_1 + Cyy_1 + \frac{D}{2}(x + x_1) + \frac{E}{2}(y + y_1) + F = 0}$$

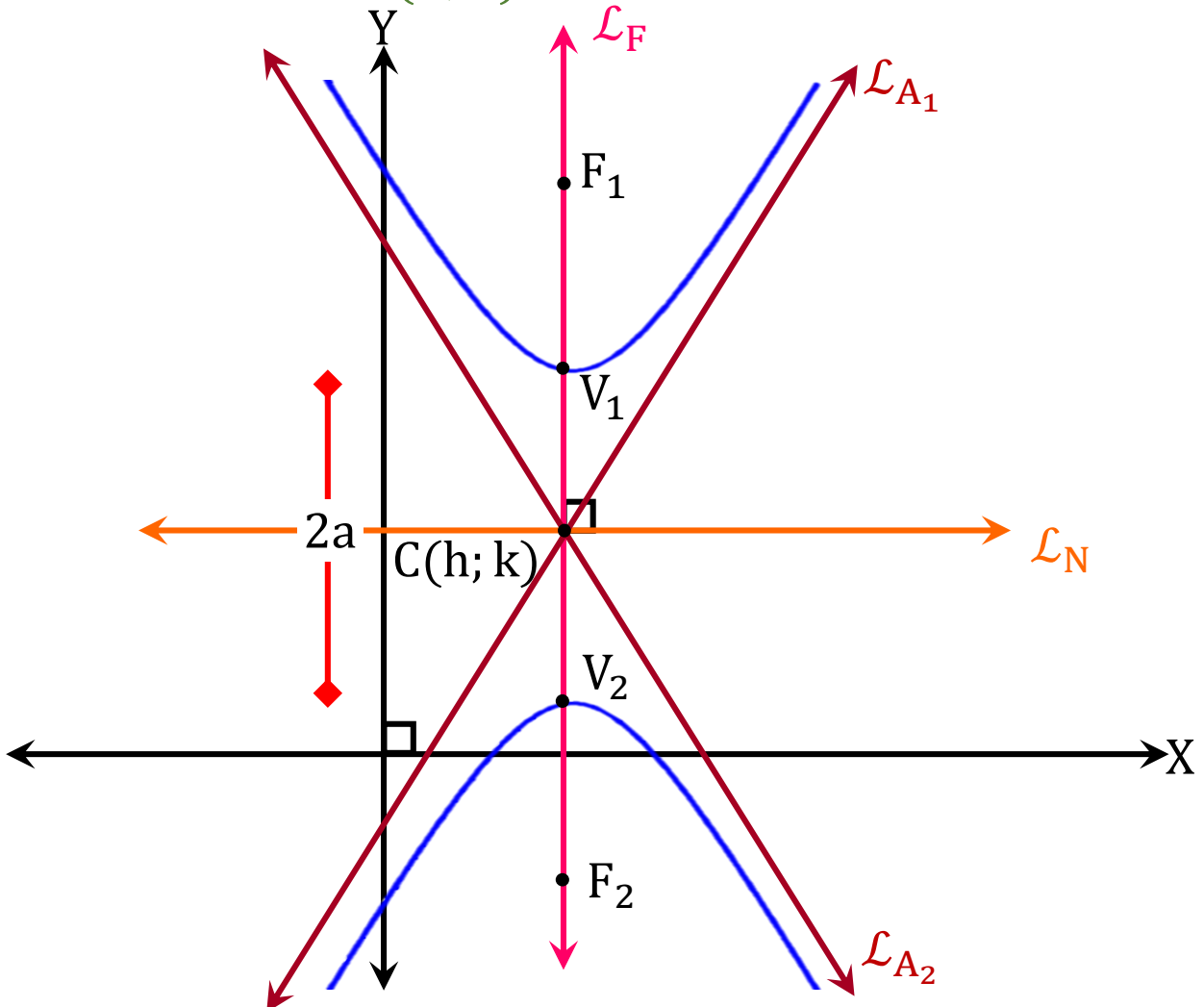
$$\mathcal{H}: \mathbf{Ax^2 + Cy^2 + Dx + Ey + F = 0}$$

HIPÉRBOLA

4.2. Eje Focal paralelo al eje Y:

a) Ecuación Ordinaria:

Centro en C(h; k)

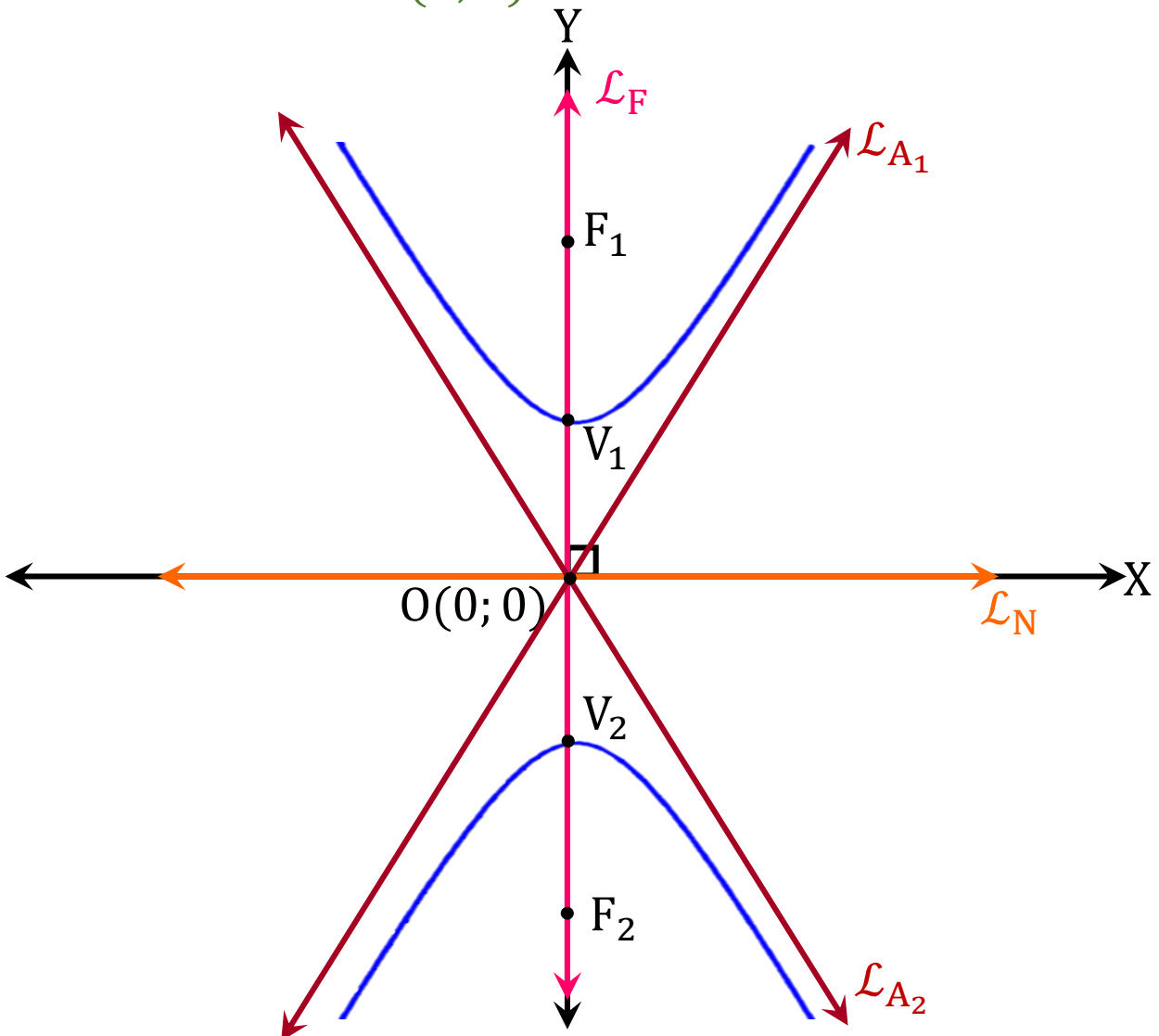


$$\mathcal{H}: \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

HIPÉRBOLA

b) Ecuación Canónica:

Centro en O(0; 0)



$$\mathcal{H}: \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

HIPÉRBOLA

c) Ecuación General:

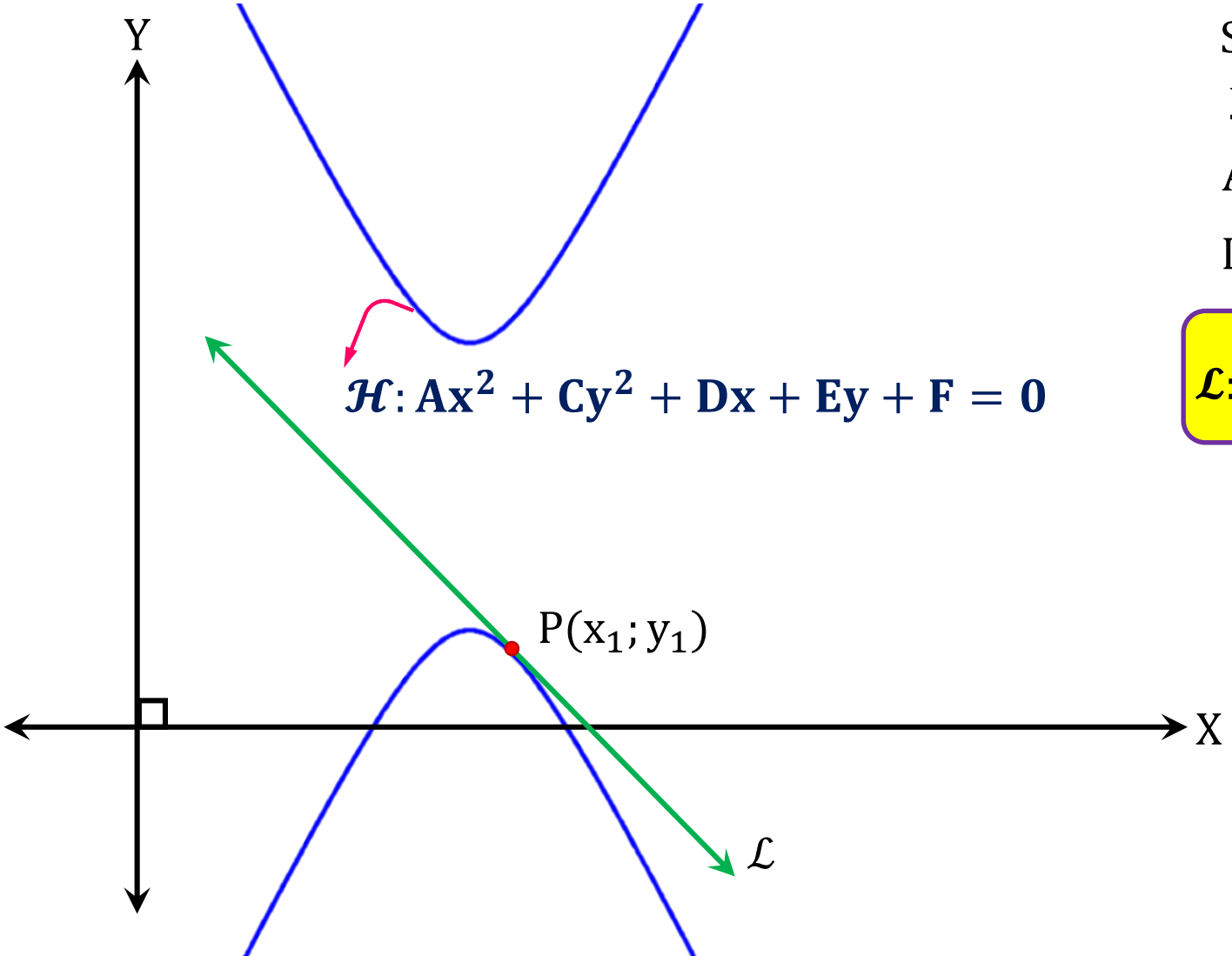
$$\mathcal{H}: Ax^2 + Cy^2 + Dx + Ey + F = 0$$

$AC < 0$

$A < 0 \ \wedge C > 0*$

HIPÉRBOLA

d) Ecuación de la Recta Tangente a la Hipérbola:



Sea $P(x_1; y_1)$ el punto de tangencia de la recta $\mathcal{L}$ y la Hipérbola:

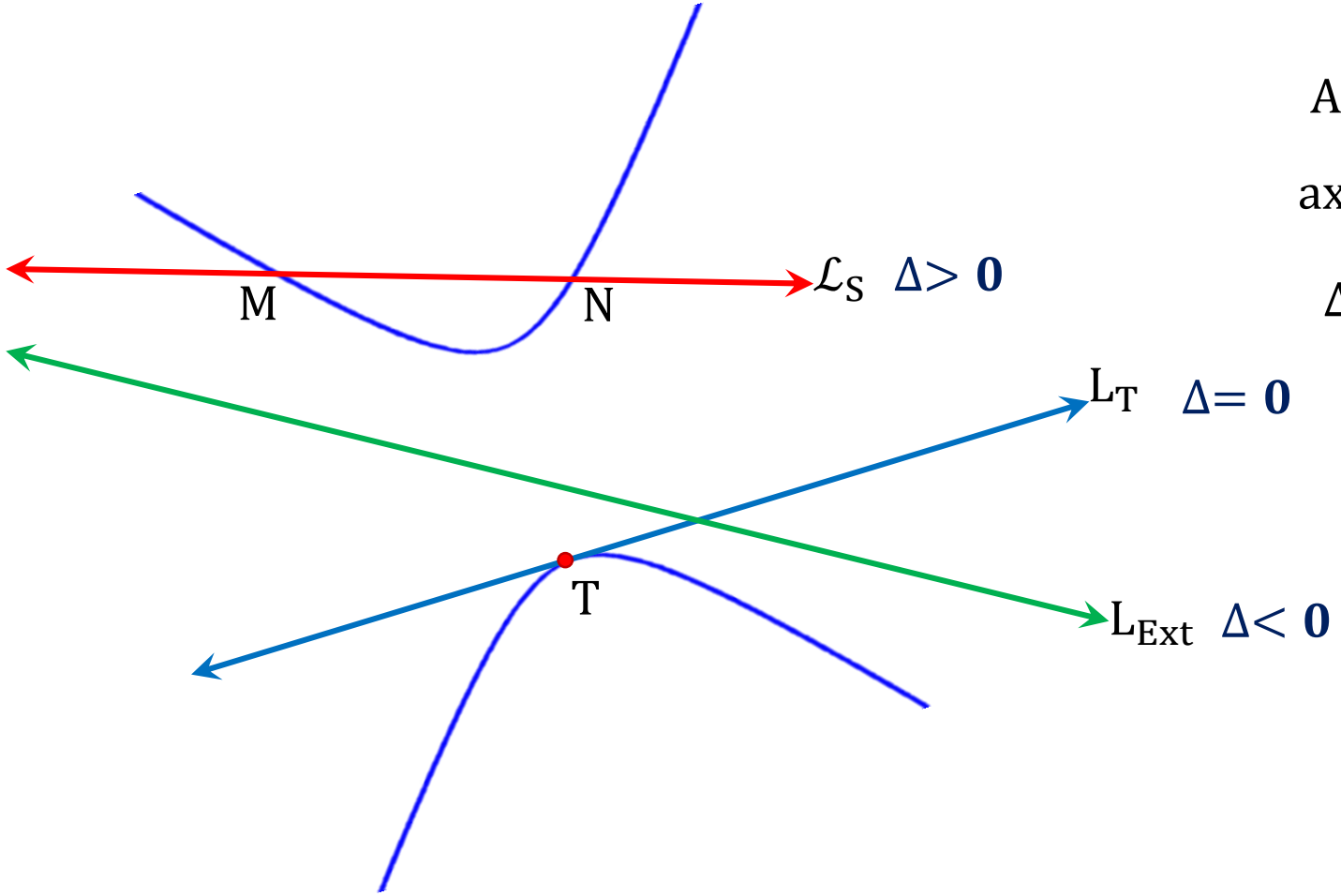
$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

La ecuación de la recta $\overleftrightarrow{L}$ es:

$$\mathcal{L}: Axx_1 + Cyy_1 + \frac{D}{2}(x + x_1) + \frac{E}{2}(y + y_1) + F = 0$$

HIPÉRBOLA

5. OBSERVACIÓN: Sea $\mathcal{H}: Ax^2 + Cy^2 + Dx + Ey + F = 0$ y la $\mathcal{L}: y = mx + b$



$$Ax^2 + C(mx + b)^2 + Dx + E(mx + b) + F = 0$$

$$ax^2 + bx + c = 0$$

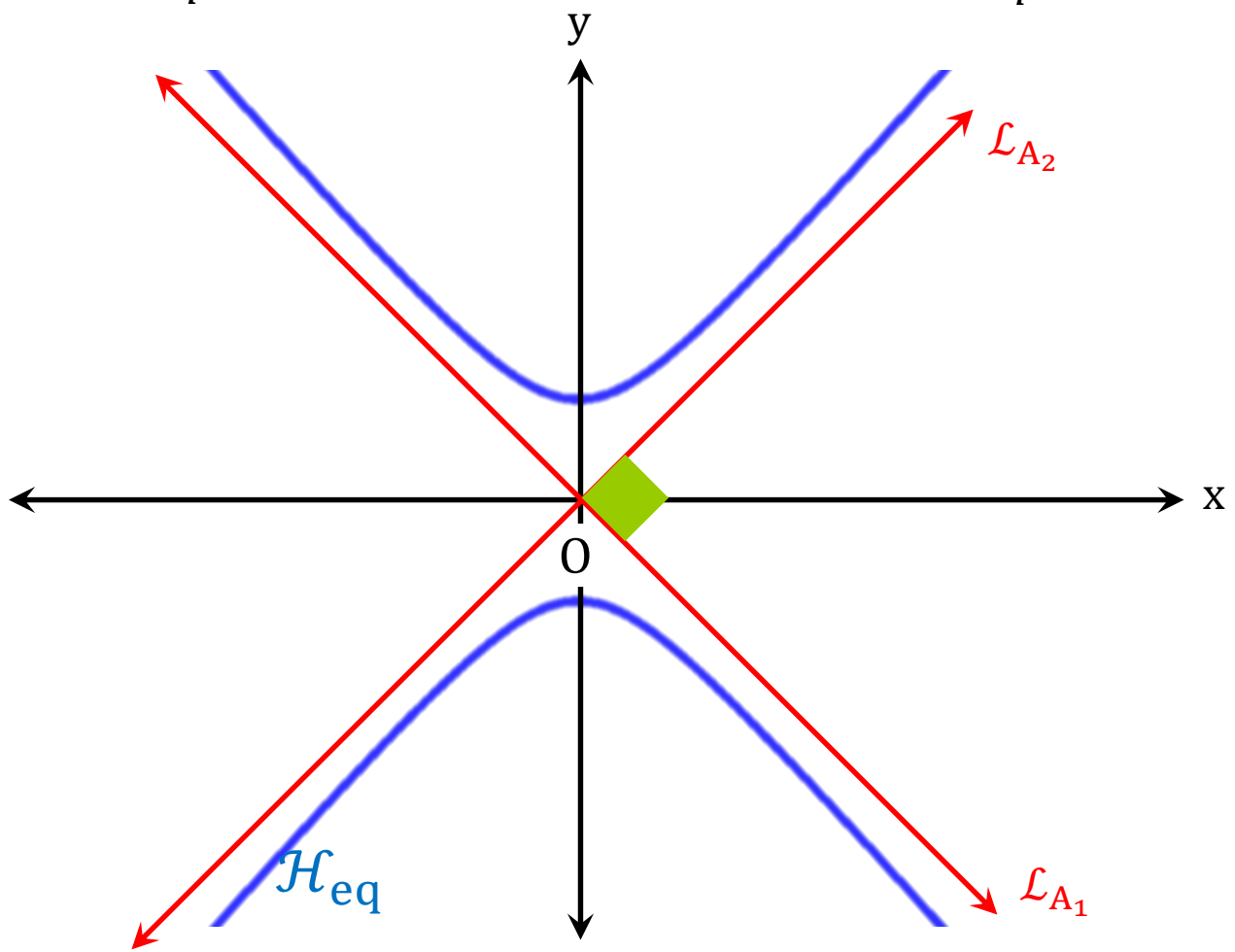
$$\Delta = b^2 - 4ac$$

HIPÉRBOLA

6. HIPÉRBOLA EQUILÁTERA:

Es aquella Hipérbola en el cual su eje transverso es igual a su eje conjugado (a=b). Su ecuación es de la forma:

$$\mathcal{H}_{Eq}: (x-h)^2 - (y-k)^2 = a^2 \quad \vee \quad \mathcal{H}_{Eq}: (y-k)^2 - (x-h)^2 = a^2$$



HIPÉRBOLA

7. HIPÉRBOLA CONJUGADAS:

Dos Hipérbolas son conjugadas si el eje transverso de cada una es idéntica al eje conjugado de la otra.

$$\mathcal{H}: \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\mathcal{H}_C: \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

$$\mathcal{H}: \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\mathcal{H}_C: \frac{(x-h)^2}{b^2} - \frac{(y-k)^2}{a^2} = 1$$

HIPÉRBOLA

8. ECUACIÓN DE LAS ASÍNTOTAS:

Sea:

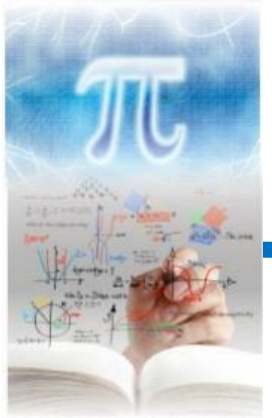
$$\mathcal{H}: \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

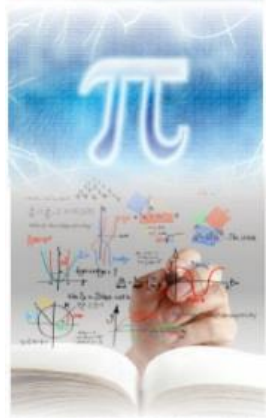
$$\mathcal{L}_{A_1\, y\, A_2}: \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 0$$

$$\mathcal{H}: \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\mathcal{L}_{A_1\, y\, A_2}: \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 0$$

MOMENTO DE PRACTICAR





PROBLEMAS Y RESOLUCIÓN

HIPÉRBOLA

11. Dada la hipérbola: $2x^2 - y^2 = 3$ ¿cuánto mide su eje transverso?

Resolución:

$$\mathcal{H}: 2x^2 - y^2 = 3$$

$$\frac{2x^2}{3} - \frac{y^2}{3} = 1$$

$$\frac{x^2}{\frac{3}{2}} - \frac{y^2}{3} = 1$$

a^2 $\quad$ b^2

$$a^2 = \frac{3}{2}$$

$$a = \sqrt{\frac{3}{2}}$$

Eje transverso:

$$V_1V_2 = 2a$$

$$V_1V_2 = 2\sqrt{\frac{3}{2}}$$

$$V_1V_2 = 2\frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

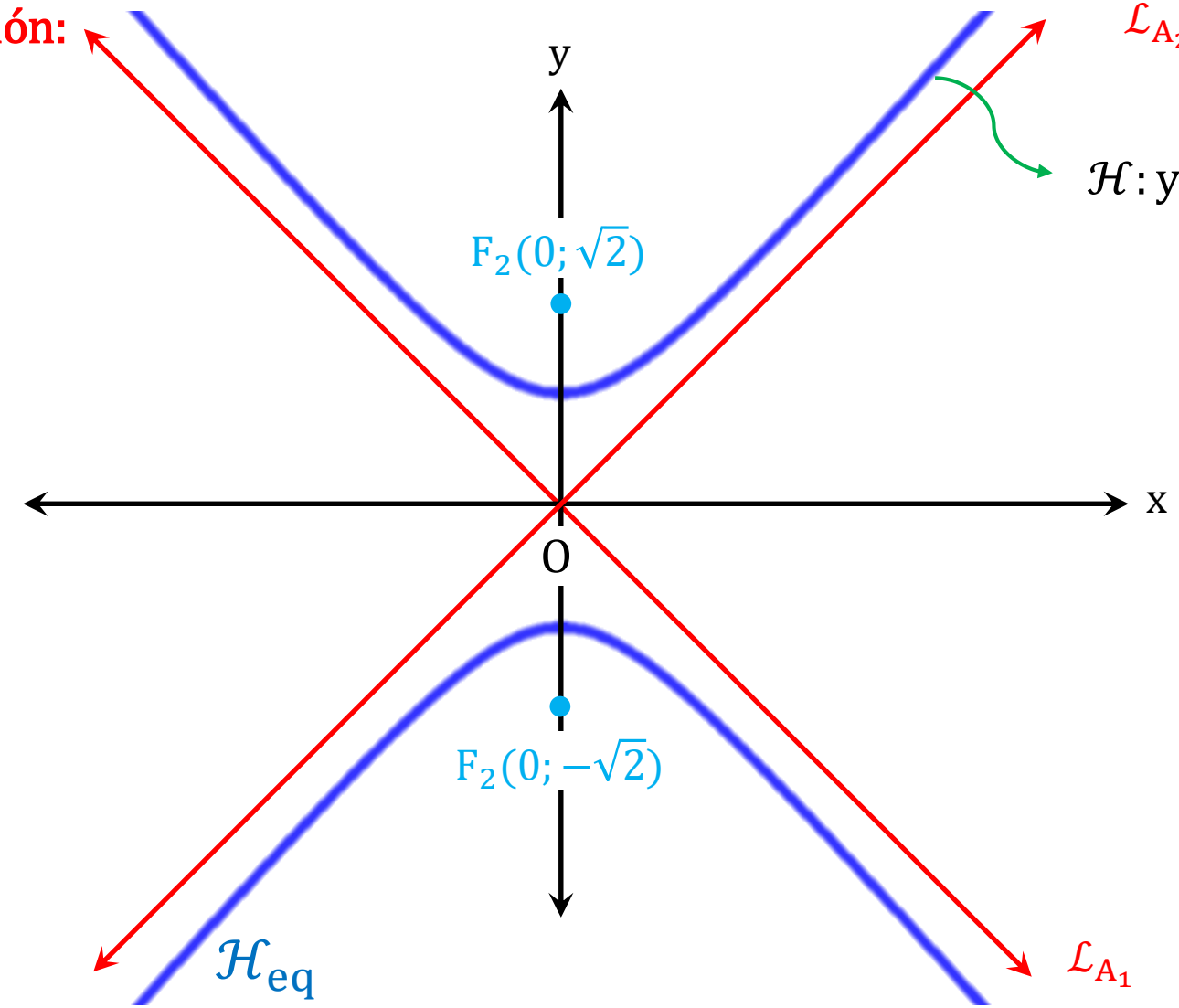
$$\therefore \boldsymbol{V_1V_2 = \sqrt{6}}$$

CLAVE: C

HIPÉRBOLA

12. Determinar la ecuación de la hipérbola equilátera que tiene su centro en el origen de coordenadas y uno de sus focos es $(0; -\sqrt{2})$

Resolución:



$\mathcal{H}: y^2 - x^2 = a^2$

$c^2 = a^2 + b^2$

a^2

$c^2 = 2a^2$

$c = a\sqrt{2}$

$\sqrt{2} = a\sqrt{2}$

$a = 1$

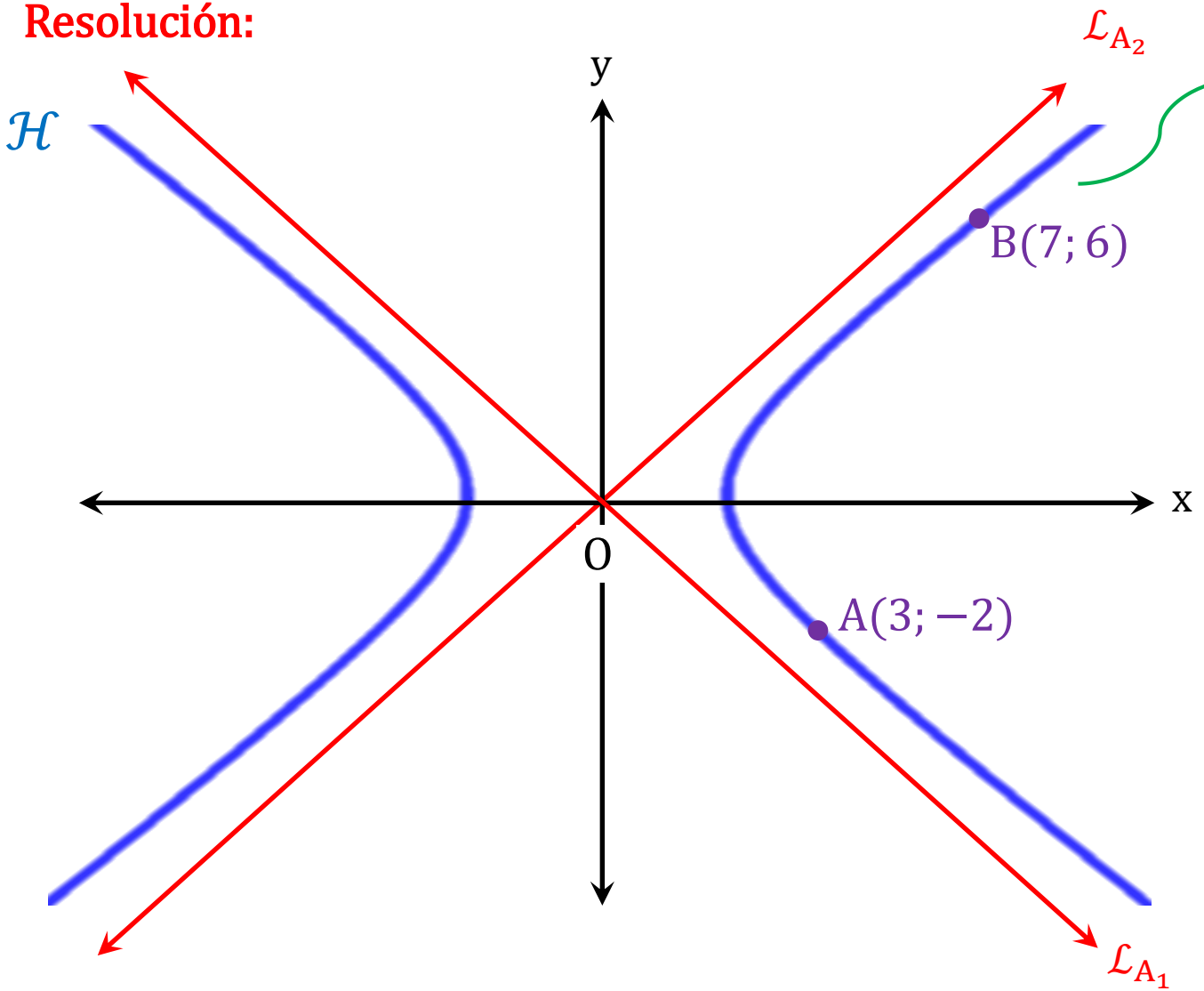
$\therefore \mathcal{H}: y^2 - x^2 = 1$

CLAVE: A

HIPÉRBOLA

13. Halle la ecuación de la hipérbola que pasa por los puntos $A(3;\ -2)$ y $B(7;\ 6)$, tiene su centro en el origen y el eje transverso coincide con el eje X.

Resolución:



$$\mathcal{H}: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$A(3;-2) \in \mathcal{H}: \ \frac{9}{a^2} - \frac{4}{b^2} = 1$$

$$B(7;6) \in \mathcal{H}: \ \frac{49}{a^2} - \frac{36}{b^2} = 1$$

$$\frac{36}{b^2} = \frac{81}{a^2} - 9 = \frac{49}{a^2} - 1$$

$$a = 2 \rightarrow b = \frac{4}{\sqrt{5}}$$

$$\mathcal{H}: \frac{x^2}{4} - \frac{5y^2}{16} = 1$$

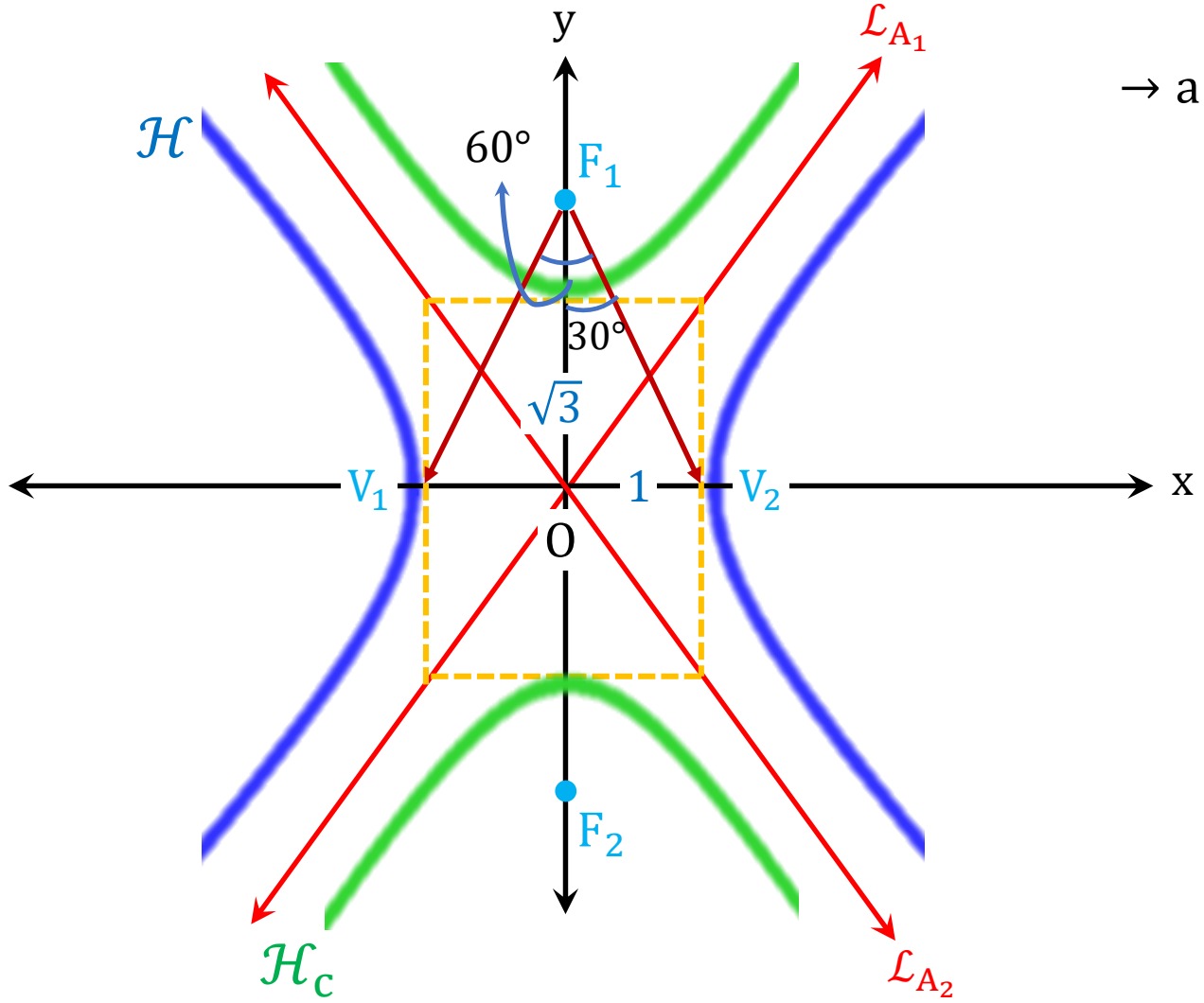
$$\therefore \mathcal{H}: 4x^2 - 5y^2 = 16$$

CLAVE: A

HIPÉRBOLA

14. Determinar la excentricidad de la hipérbola si el segmento comprendido entre sus vértices se ve desde los focos de la hipérbola conjugada bajo un ángulo de 60°

Resolución:



$\rightarrow a = 1 \wedge \quad c = \sqrt{3}$

$$e = \frac{c}{a}$$

$$e = \frac{\sqrt{3}}{1}$$

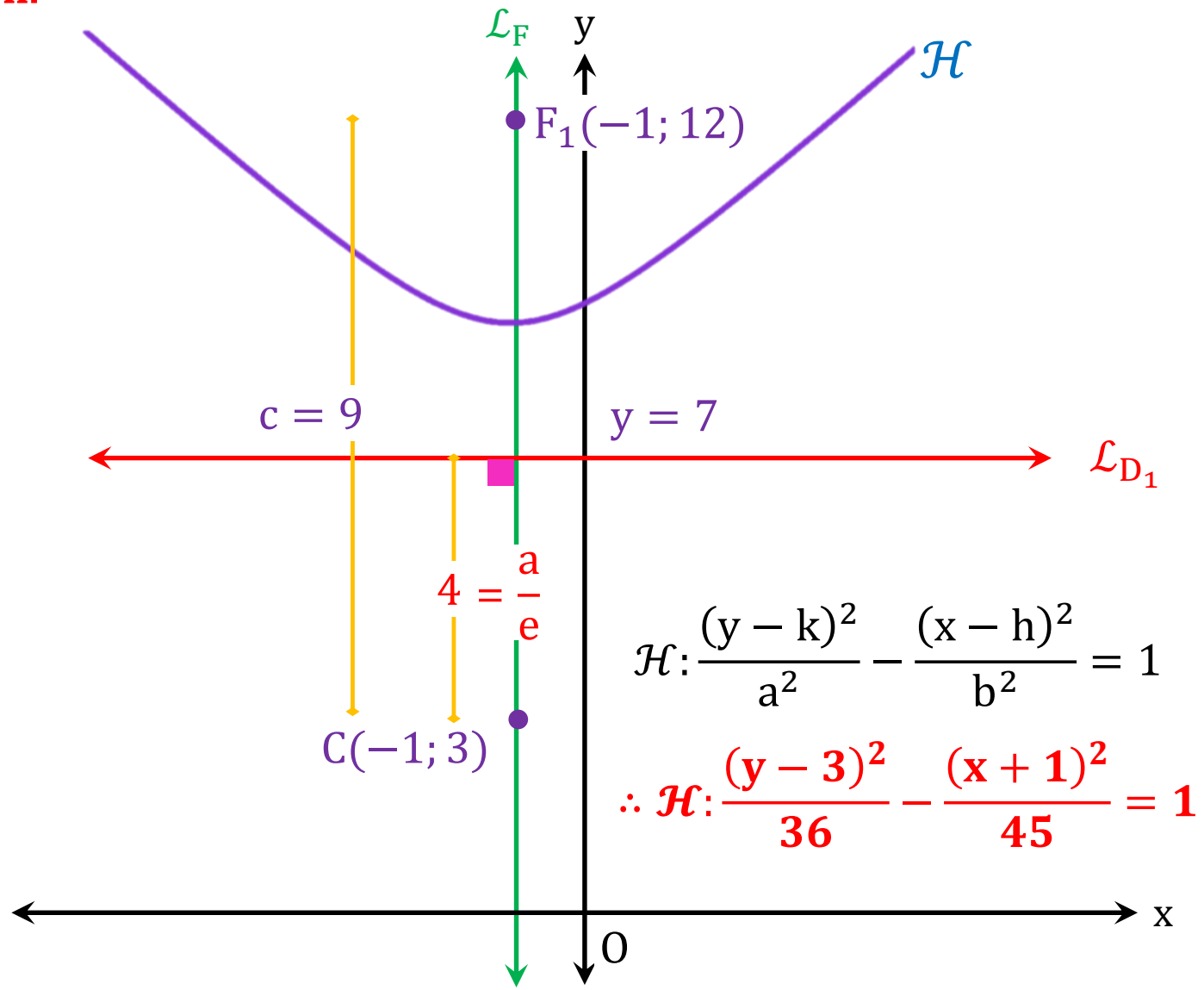
$\therefore \mathbf{e = \sqrt{3}}$

CLAVE: B

HIPÉRBOLA

15. Hallar la ecuación de una hipérbola de centro $(-1; 3)$, distancia focal igual a 18 y una de sus directrices: $y = 7$.

Resolución:



$$\frac{a}{e} = 4$$

$$\frac{a}{\frac{c}{a}} = 4$$

$$\frac{a^2}{9} = 4$$

$$a^2 = 36$$

$$a = 6$$

$$c^2 = a^2 + b^2$$

$$81 = 36 + b^2$$

$$b^2 = 45$$

$$\mathcal{H}: \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

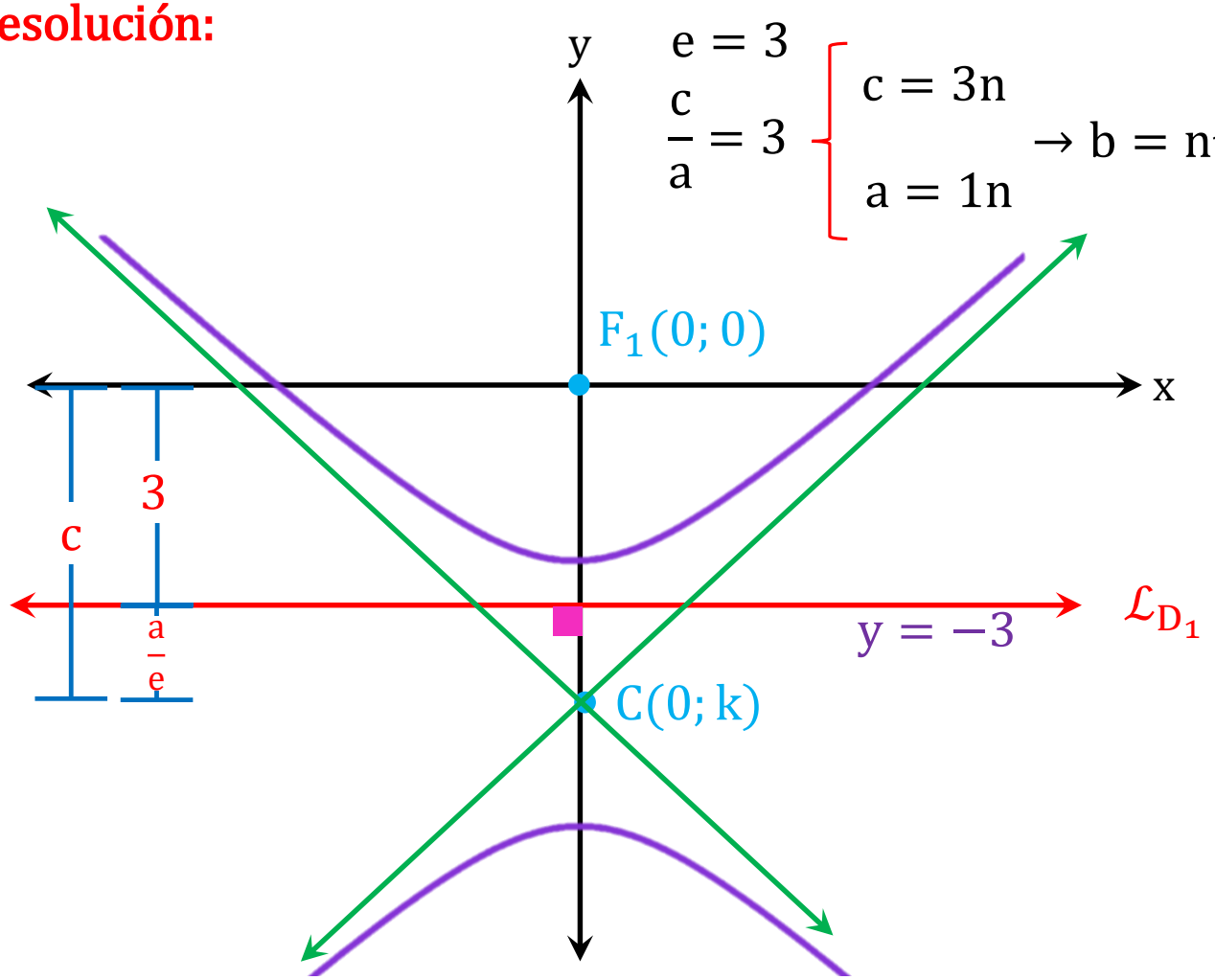
$$\therefore \mathcal{H}: \frac{(y-3)^2}{36} - \frac{(x+1)^2}{45} = 1$$

CLAVE: C

HIPÉRBOLA

16. Determine la ecuación de la hipérbola, cuya excentricidad es 3, uno de sus focos es el origen de coordenadas y la directriz que le corresponde es la recta $y = -3$

Resolución:



$$\frac{e = 3}{\frac{c}{a} = 3} \begin{cases} c = 3n \\ a = 1n \end{cases} \rightarrow b = n\sqrt{8}$$

$$c = \frac{27}{8} \rightarrow C\left(0; -\frac{27}{8}\right)$$

$$a = \frac{9}{8} \qquad b = \frac{9}{8}\sqrt{8}$$

$$\rightarrow c = 3 + \frac{a}{e}$$

$$3n = 3 + \frac{n}{3}$$

$$\frac{8n}{3} = 3$$

$$n = \frac{9}{8}$$

$$\mathcal{H}: \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

HIPÉRBOLA

$$\mathcal{H}: \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\mathcal{H}: \frac{\left(y-\left(-\frac{27}{8}\right)\right)^2}{\left(\frac{9}{8}\right)^2} - \frac{(x-0)^2}{\left(\frac{9}{8}\sqrt{8}\right)^2} = 1$$

$$\mathcal{H}: \frac{\left(y+\frac{27}{8}\right)^2}{\frac{81}{64}} - \frac{x^2}{\frac{81}{8}} = 1$$

$$\mathcal{H}: 64\left(y+\frac{27}{8}\right)^2 - 8x^2 = 81$$

$$\mathcal{H}: 64\left(y^2+\frac{27}{4}y+\frac{729}{64}\right) - 8x^2 = 81$$

$$\mathcal{H}: 64y^2 + 16.27y + 729 - 8x^2 = 81$$

$$\mathcal{H}: 64y^2 - 8x^2 + 16.27y + 648 = 0$$

$$\therefore \mathcal{H}: \mathbf{8y^2 - x^2 + 54y + 81 = 0}$$

CLAVE: C

HIPÉRBOLA

17. Hallar los puntos de intersección de la recta: $4x - 3y - 16 = 0$ y la hipérbola: $\frac{x^2}{25} - \frac{y^2}{16} = 1$

Resolución:

$$\mathcal{H}: \frac{x^2}{25} - \frac{y^2}{16} = 1 \qquad \mathcal{L}: 4x - 3y - 16 = 0$$

$$\mathcal{H}: 16x^2 - 25y^2 = 400 \qquad \mathcal{L}: 4x = 3y + 16$$

$$(3y + 16)^2 - 25y^2 = 400$$

$$9y^2 + 96y + 256 - 25y^2 = 400$$

$$16y^2 - 96y + 144 = 0$$

$$y^2 - 6y + 9 = 0$$

$$(y - 3)^2 = 0$$

$$y = 3 \rightarrow x = \frac{25}{4}$$

$$\therefore \mathbf{T}\left(\frac{25}{4}; 3\right)$$

CLAVE: B

HIPÉRBOLA

18. Dada la ecuación de la hipérbola $\frac{(x-4)^2}{16}-\frac{y^2}{128}=1$ encontrar las ecuaciones de las directrices y asíntotas.

Resolución:

$$\mathcal{H}: \frac{(x-4)^2}{16}-\frac{y^2}{128}=1$$

$$\mathcal{L}_{A_1 \text{ y } A_2}: \frac{(x-4)^2}{16}-\frac{y^2}{128}=0$$

$$\frac{(x-4)^2}{16}=\frac{y^2}{128}$$

$$(x-4)^2=\frac{y^2}{8}$$

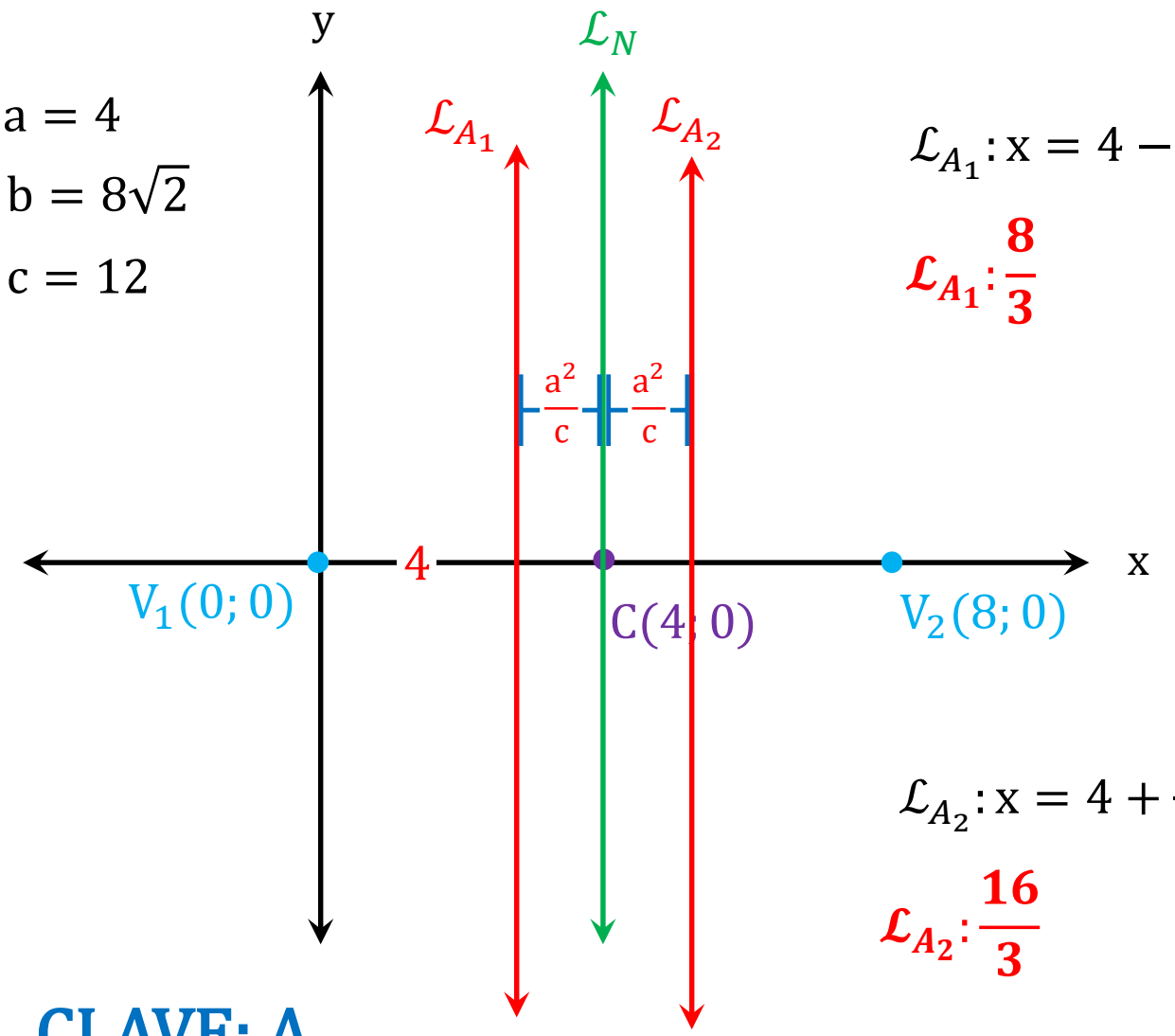
$$|x-4|=\left|\frac{y}{2\sqrt{2}}\right|$$

$$\mathbf{y=\pm 2\sqrt{2}(x-4)}$$

$a=4$

$b=8\sqrt{2}$

$c=12$



$$\mathcal{L}_{A_1}: x=4-\frac{4^2}{12}$$

$$\mathcal{L}_{A_1}: \mathbf{\frac{8}{3}}$$

$$\mathcal{L}_{A_2}: x=4+\frac{4^2}{12}$$

$$\mathcal{L}_{A_2}: \mathbf{\frac{16}{3}}$$

CLAVE: A

HIPÉRBOLA

19. Calcule el baricentro de la región triangular formada por los vértices de la hipérbola:

$\mathcal{H}: 9x^2 - 4y^2 + 54x + 16y + 29 = 0$ Y el punto $A(0; -5)$

Resolución:

$$\mathcal{H}: 9x^2 + 54x - 4y^2 + 16y = -29$$

$$\mathcal{H}: 9(x^2 + 6x + 9) - 4(y^2 - 4y + 4) = -29 + 81 - 16$$

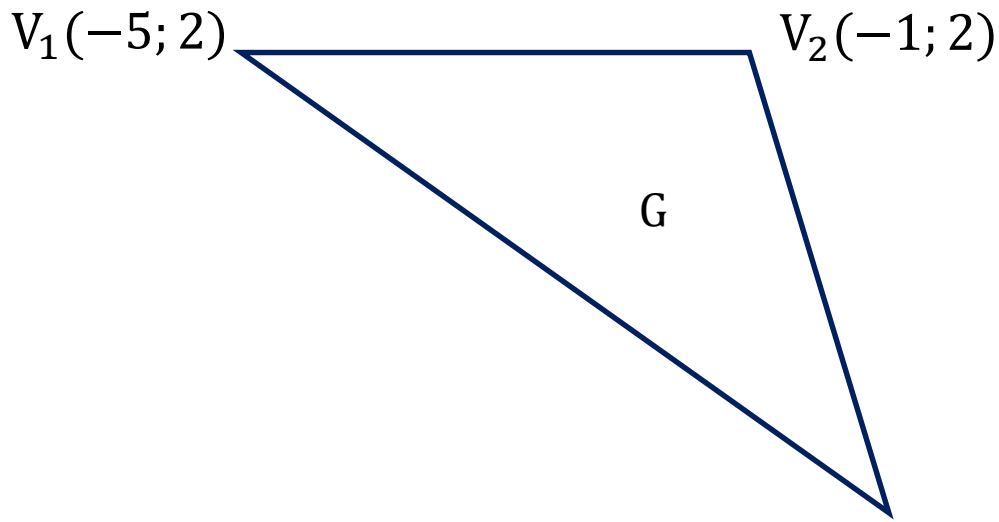
$$\mathcal{H}: 9(x+3)^2 - 4(y-2)^2 = 36$$

$$\mathcal{H}: \frac{(x+3)^2}{4} - \frac{(y-2)^2}{9} = 1$$

$C(-3; 2)$ $a = 2$ y $b = 3$

Hipérbola: Horizontal

$V_1(-5; 2)$ $V_2(-1; 2)$



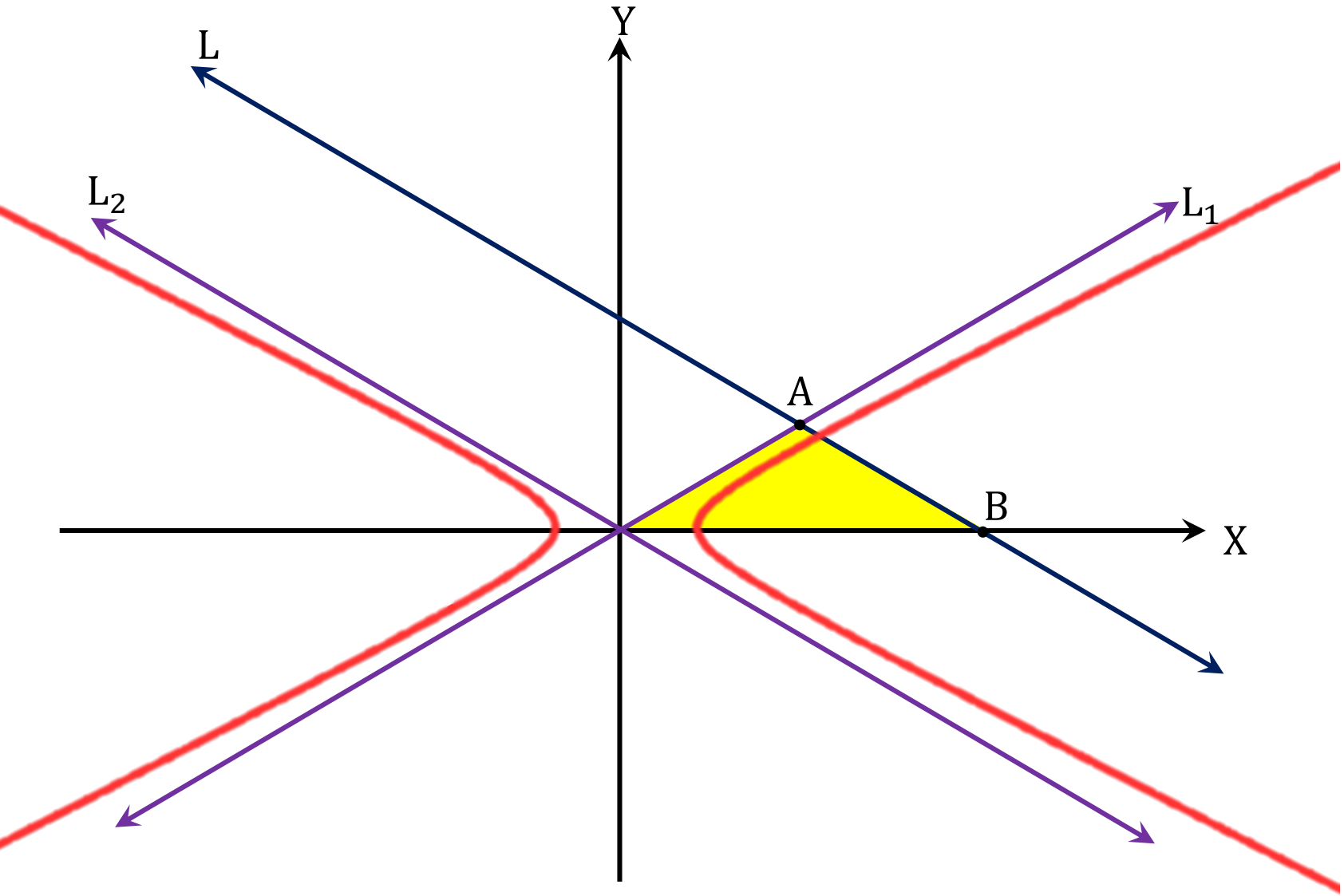
$$G\left(\frac{-5-1+0}{3} : \frac{2+2-5}{3}\right)$$

$$\therefore \mathbf{G\left(-2; -\frac{1}{3}\right)}$$

CLAVE: B

HIPÉRBOLA

20. Si L_1 y L_2 son asíntotas de la hipérbola $\mathcal{H}: x^2 - 4y^2 = 16$ y $L: \frac{y}{2} + \frac{x}{5} = 1$, entonces el área de la región sombreada es:



Resolución:

$$\mathcal{H}: x^2 - 4y^2 = 16$$

$$\mathcal{H}: \frac{x^2}{16} - \frac{y^2}{4} = 1$$

$$\mathcal{L}_{A_1 \text{ y } A_2}: \frac{x^2}{16} - \frac{y^2}{4} = 0$$

$$\frac{x^2}{16} = \frac{y^2}{4}$$

$$\frac{x^2}{4} = \frac{y^2}{1}$$

$$y = \pm\frac{x}{2}$$

$$\mathcal{L}_1: y = \frac{x}{2}$$

ELIPSE

$A: \mathcal{L}_1 \cap \mathcal{L} \rightarrow \mathcal{L}_1 = \mathcal{L}$

en $\mathcal{L}$: $\frac{y}{2} + \frac{x}{5} = 1$ $\xrightarrow{\text{Si: } y = 0}$ en $\mathcal{L}$: $\frac{y}{2} + \frac{x}{5} = 1$

$\frac{y}{2} + \frac{2y}{5} = 1$

$0 + \frac{x}{5} = 1$

$\frac{9y}{10} = 1$

$x = 5$

$y = \frac{10}{9}$

Altura

Base

$S = \frac{5 \times \frac{10}{9}}{2}$

$\therefore \mathbf{S = \frac{25}{9}}$

CLAVE: B

FIN DE LA SESIÓN

PRACTICA Y APRENDERÁS